Whence the Mathematical Universe? [Collapse.rev.12.6.doc]

Adam InTae Gerard

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adam.gerard@gmail.com

Whence the Mathematical Universe?

Abstract: Tegmark has recently defended his so-called Mathematical Universe Hypothesis (MUH) and has maintained that ultimately "our successful [scientific] theories are not mathematics approximating physics, but mathematics approximating mathematics."[1] If true, the MUH recommends nothing short of a (meta-)physical revolution. However, this proposed Theory of Everything (TOE) has met with a strongly divided mixture of praise and condemnation in both philosophical and scientific circles.

If the reader will indulge me, I will in the first half of this essay attempt to explore Tegmark's basic assumptions, connecting these ideas to current trends in the philosophy of science, mathematics and metaphysics in order to investigate this novel and innovative theory. Having then uncovered Tegmark's underlying commitments, the validity of the MUH can be accurately assessed.

In the second section of this paper I will offer a defense of a more limited metaphysical view motivated by Tegmark's MUH.

Index of Sections:

Word Count: 17,899

[1] (Tegmark, 4)

1. The Mathematical Universe Hypothesis

Tegmark has recently advanced the MUH as a proposed TOE asserting that "our successful [scientific] theories are not mathematics approximating physics, but mathematics approximating mathematics"[2] and that "the universe *is* a mathematical structure."[3]

While this may at first sight appear radical, Tegmark reminds us that "The idea that our universe is in some sense mathematical goes back at least to the Pythagoreans… Galileo Galilei stated that the Universe is a grand book written in the language of mathematics."[4]

He appears motivated to this position by Wigner's famous observation regarding the 'unreasonable effectiveness'[5] and indispensability of mathematics toward science:

> "In his above-mentioned 1967 essay, Wigner argued that "the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious", and that "there is no rational explanation for it". The MUH provides this missing explanation. It explains the utility of mathematics for describing the physical world as a natural consequence of the fact that the latter is a mathematical structure, and we are simply uncovering this bit by bit."[6]
>
> "After Wigner had written his 1967 essay, the standard model of particle physics revealed new "unreasonable" mathematical order in the microcosm of elementary particles and in the macrocosm of the early universe. I know of no other compelling explanation for this trend than that the physical world really is completely mathematical."[7]

Indeed, "... the problem is really an old one, and there are old solutions too. Pythagoras claimed that there was no distinction between the world of physics and the world of mathematics. Plato argued there was a very great difference: it amounted to concrete versus abstract, a distinction denied by Pythagoras. For Plato the concrete world instantiated (or `partook of') the abstract forms (albeit imperfectly). This Platonic account is somewhat similar to the model-based view... More recently structural realists have answered the question by arguing that science is about the discovery of structural aspects of the world, and these structural aspects are essentially mathematical. Radical `ontic' structural realists turn this in to an ontological claim: science is about structure and structure is all there is."[8]

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[2] (Tegmark, 5)
[3] (Ibid.)
[4] (Tegmark, 1)
[5] see (Wigner)
[6] (Tegmark, 4)
[7] (Ibid.)
[8] (Rickles, 1-2)

Additionally, when we look to contemporary scientific practice we find that working physicists often refer to their scientific models and theories as being ‘physico-mathematical models’ and may talk about their various mathematical models as ‘being isomorphic to the world’ (and also many philosophers of science).[9] Additionally, Pythagoreanism seems to be a current guiding assumption in the investigation of the physical world :

> “We have observed that the heart of the new scientific metaphysics is to be found in the ascription of ultimate reality and causal efficacy to the world of mathematics… the real world in which man lives is no longer regarded as a world of substances possessed of as many ultimate qualities as can be experienced in them, but has become a world of atoms (now electrons), equipped with none but mathematical characteristics and moving according to laws fully statable in mathematical form.”[10]

Indeed, this trend can be traced back to Newton. Recall his famous maxim *hypotheses non fingo*[11] regarding any epistemological or metaphysical objections that might be raised against the *Principia* and his mathematical formalization of gravity.

Prima facie it would appear that Tegmark’s MUH has the tacit support of a particular intellectual current in the philosophy of science. It is at this point that I feel it prudent, however, to note that Tegmark relies on a number of philosphical assumptions that are highly contentious at the present moment. First, for Tegmark’s MUH to succeed it would have to be the case that we adopt scientific realism[12] and with the condition that such a scientific realism be supported by a form of Ontic Structuralism.[13]

Secondly, Tegmark (and those who have espoused Ontic Structuralism) appear committed to a particular view of just what a scientific theory amounts to: the semantic view of scientific theories. Additionally, Tegmark’s particular commitment to this view

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[9] consider “We may then say that the theory is *true* if it is isomorphic to the world, and *empirically adequate* if some of its substructures are isomorphic to the observable part of the world” from (Suárez [1], 2); I note that Suárez seems to prefer a view of scientific representation that is not reliant on isomorphism.

[10] (Burtt, 303)

[11] I feign no hypothesis

[12] Which I infer from his commitment to his so-called External Reality Hypothesis (ERH); (Tegmark, 1)

[13] He states “In the philosophy literature, the name “structural realism” has been coined for the doctrine that the physical domain of a true theory corresponds to a mathematical structure, and the name “universal structural realism” has been used for the hypothesis that the physical universe is isomorphic to a mathematical structure.” (Tegmark, 3); also, “If one rejects the ERH, one could argue that our universe is somehow made of stuff perfectly described by a mathematical structure, but which also has other properties that are not described by it, and cannot be described in an abstract baggage-free way. This viewpoint, corresponding to the “epistemic” version of universal structural realism in the philosophical terminology of [Ladyman and McCabe], would make Karl Popper turn in his grave, since those additional bells and whistles that make the universe nonmathematical by definition have no observable effects whatsoever.” (Tegmark, 4)

seems to rely on an isomorphic theory of representation.[14] In order to realize his claim that "the world *is* a mathematical structure" it would also appear that Tegmark is explicitly committed to a version of mathematical Platonism[15], of the *ante rem* structuralist variety recently espoused by Resnik and Shapiro (among others). I will note that Tegmark's position gives some flexibility as to whether mathematical structures are to be understood as per the so-called *plenitudinous*[16] Platonist position ("the central thesis of this theory is that every logically consistent mathematical theory *necessarily* refers to an abstract entity"[17] or "It is the thesis that any mathematical object which *can* exist, *does* exist."[18]) or to a more restricted "computable"[19] form of Platonism.

Thirdly, Tegmark explicitly commits to modal realism as a way to defeat the *problem of contingency* (crassly, why some but not all?).[20] He also appears to endorse both the Many Worlds Interpretation (MWI), as a way to overcome the *problem of measurement*, and theories of cosmological symmetry breaking in order to deal with the 'fine-tuned' nature of the universe.

I have neither the resources or the space to investigate the scientific and philosophical controversy surrounding the ontological status of *possibilia*, modal realism or the MWI. For our present purposes, it should, however, suffice to examine Tegmark's realism, structuralism and Platonism (and how these notions relate).

In this way, the veracity of the MUH can perhaps be put as follows:

[1.0] Ontic Structuralism (is true)
[1.1] *Ante rem* structuralism (is true)
[1.2] The Isomorphic Theory of Representation (is true)

[C] MUH (is true)

I will now sketch out some of the territory we will be exploring in the remainder of this essay. Worrall articulated structural realism as a way to synthesize a response to "arguably the two most compelling arguments around"[21] - "the *no miracles* argument, and the *pessimistic meta-induction*"[22]- regarding the debate over realism in the philosophy of science. This view is a revival of positions that trace their lineage back to

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[14] "We write is rather than corresponds to here, because if two structures are isomorphic, then there is no meaningful sense in which they are not one and the same… If our external physical reality is isomorphic to a mathematical structure, it therefore fits the definition of being a mathematical structure." (Tegmark, 4)

[15] He states that for the MUH to be true "mathematical structures need to be to be real" (Tegmark, 22) presumably in an ontological sense.

[16] what has been also dubbed *Full-Blooded* Platonism; see (Balaguer)

[17] (Horsten)

[18] (Restall, 82)

[19] see (Tegmark, 20-4)

[20] (Tegmark, 17)

[21] (Ladyman**,** 409)

[22] my emphasis; (Ibid.)

Russell, Cassirer and Poincaré. Two forms of this structuralism tend to dominate the contemporary field: epistemic structural realism[23] (ESR), is the view that we should commit ourselves only to the mathematical content of, and not to the objects talked about by, scientific theories and the more radical view, ontic structural realism (OR), which denies the epistemic limitations posed by ESR by asserting that *structures are all that there are***.**

Meanwhile, structuralism has acquired advocates in the philosophy of mathematics. Shapiro has advocated *ante rem* mathematical structuralism (ARS) to resolve difficult problems facing the mathematical realist. Opponents of this view have taken on the mantle of *in re* categorical structuralism (IRS).

2. Structure

Structure is a basic concept. A structure is some collection of objects organized into a pattern or arrangement: a group of bricks are arranged into a building, small pebbles are placed in a pile. Intuitively, a structure is a set of objects with a set of relations over those objects. This basic concept has powerful applications. In mathematics, the notion of structure has been extended into many formal systems through model theory. For example the sentence '1+1=2' is *satisfied by*[24] the model theoretic structure $\{|\mathbf{N}|, +, 0, <, \times\}$[25] which is characterized by the axioms of Peano arithmetic. This structure is a model for arithmetical statements where **N**[26] is the *domain of discourse*[27] over which the various relations $\{+, <, \times\}$ operate.

From this basic introduction we will attempt to flesh out the notion and defining characteristics of structure. It will be necessary, in order to give a rigorous analysis of these concepts, to delve further into the central, and essential, components of model theory. This requires, in turn, for us to investigate the zero- and first-order predicate logics - familiar territory for philosophers - which will allow us to proficiently enter into the debate surrounding structuralism in the philosophy of science and mathematics.

In general, logic is the science of truth – how truth values can be derived from other truth values using various inference rules and deduction. A zero-order logic deals in propositions or declarative sentences which can be assigned a truth value (in our present case, we will be considering binary truth valued systems only). For our purposes we will employ a simple propositional calculus of the axiomatic form developed by Łukasiewicz.[28]

Briefly, a "language is determined by its symbols along with its syntactic formation rules. A calculus is a language together with axioms and/or rules of inference for making

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[23] Worrall's original position has been referred to by this moniker.
[24] or *is true in*
[25] (Doets, 1)
[26] or $\{1, 2, 3, \ldots\}$
[27] or *the objects we are discussing* as in the example of bricks or pebbles above
[28] see (Ó Dúnlaing)

deductions within the formal language. A logic is a language along with a semantics to interpret that language."[29] We take our *propositional calculus* (alternatively called a *sentential calculus*) to be called L_1. We define L_1 as per the following:

[2.0] $L_1 = \{A, Z, I, \Omega\}$

[2.1] A is a (presumably) finite set of *propositional variables* (alternatively: *sentence letters*), such that $A = \{A_0^0, A_1^0, \ldots, B_0^0, B_1^0, \ldots, \ldots, Z_0^0, Z_1^0, \ldots\}$

[2.2] Z is the zeta set of *inference rules* valid in L_1. We will here include only the *modus ponens*, such that $Z = \{p, p \rightarrow q \vdash q\}$

[2.3] I is the set of *axiom schemata* for our logic L_1, such that:
[2.3] $I = AS1 \cup AS2 \cup AS3$
[2.4] $AS1 = \{A \rightarrow (B \rightarrow A)\}$
[2.5] $AS2 = \{(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))\}$
[2.6] $AS3 = \{(\neg A \rightarrow \neg B) \rightarrow (B \rightarrow A)\}$

[2.7] Ω is the set of primitive *logical operators* (*logical connectives*) for L_1, such that:
[2.8] $\Omega = \Omega_1 \cup \Omega_2$
[2.9] Where Ω_1 is the set of logical connectives of *arity* 1, such that $\Omega_1 = \{\neg\}$[30]
[2.10] Where Ω_2 is the set of logical connectives of *arity* 2, such that $\Omega_2 = \{\rightarrow\}$[31][32]

[2.11] The *well-formed formulae* (wff) of L_1 are recursively defined as follows:
[2.12] Any δ, where δ is a propositional variable of L_1, is a formula.
[2.13] If δ is a formula then, $\neg\delta$ is a formula.
[2.14] If δ and φ[33] are formulas then, $\delta \rightarrow \varphi$ is a formula.
[2.15] There are no other wff.

[*] We follow standard convention regarding parenthetical dropping, quotation and uniform substitution.

We now expand on L_1 by adding the basic apparatus of a first-order[34] calculus. We will call our first-order calculus L_2. Simply, we can define L_2 by adding the following to L_1:

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[29] his emphasis; (Mattison, 1)

[30] The truth table for this connective is:

A	¬A
T	F
F	T

[31] The truth table for this connective is:

A	B	A → B
T	T	T
T	F	F
F	F	T
F	T	T

[32] Three auxiliary definitions take us to the familiar set of logical connective $\{\wedge, \vee, \neg, \rightarrow, \leftrightarrow\}$:
(i) $p \vee q :_{df} \neg p \rightarrow q$ (ii) $p \wedge q :_{df} \neg(p \rightarrow \neg q)$ (iii) $p \leftrightarrow q :_{df} \neg((p \rightarrow q) \rightarrow \neg(q \rightarrow p))$

[33] δ and φ are *meta-variables* which range over the expressions of L_1.

variable, *quantifier*, *relation*, and *function* (*operation*) symbols along with additional axiom schemata, rules of inference and wff formation rules.

For brevity's sake, we will only treat some of these basic notions here:

[2.16] *Arity* is the number of arguments that a relation or a function can take.
[2.17] The *relation*[35] symbols in the *alphabet* (*vocabulary*) of L_2 comprise a set of n-ary relation symbols usually denoted as per the following: $\{R_0^0, R_1^0, \ldots, R_0^1, R_1^1, \ldots, R_0^2, R_1^2, \ldots, \ldots, R_0^n, R_1^n, \ldots\}$
[2.18] A *predicate* is an unary relation.
[2.19] A sentence letter is a zero-place (zero-arity) relation symbol.
[2.20] The *function*[36] symbols in the alphabet of L_2 comprise a set of n-ary function symbols usually denoted as per the following: $\{f_0^0, f_1^0, \ldots, f_0^1, f_1^1, \ldots, f_0^2, f_0^2, \ldots, \ldots, f_0^n, f_1^n, \ldots\}$
[2.21] Where a *constant* symbol is a zero-place (zero-arity) function symbol.
[2.22] The *variable*[37] symbols in the alphabet of L_2 comprise a set of variable symbols usually denoted as per the following: $\{n_0, n_1, \ldots, o_0, o_1, \ldots, \ldots, z_0, z_1, \ldots\}$

[2.23] The set of *quantifier* symbols in the alphabet of L_2 is as follows: $\{\exists\}$[38]

[2.24] We add two axiom schemata to the set axiom of schemata in L_1:
[2.25] $AS4 = \{\forall x\, F(x) \rightarrow F(y)\}$
[2.26] $AS5 = \{F(y) \rightarrow \exists x\, F(x)\}$
[2.27] Where $F(x)$ is any sentential formula in which x occurs free, y is a term, $F(y)$ is the result of substituting y for the free occurrences of x in sentential formula F, and all occurrences of all variables in y are free in F.[39]

[2.29] We join *modus ponens* with two additional rules of inference.
[2.30] Z is the zeta set of inference rules valid in L_2, such that:
[2.31] $Z = MP \cup I_1 \cup I_2$
[2.32] $MP = \{p, p \rightarrow q \vdash q\}$
[2.33] $I_1 = \{G \rightarrow F(x) \vdash G \rightarrow \forall x\, F(x)\}$
[2.34] $I_2 = \{F(x) \rightarrow G \vdash \exists x\, F(x) \rightarrow G\}$
[2.35] Where $F(x)$ is any sentential formula in which x occurs as a free variable and x does not occur as a free variable in formula G.[40]

[34] A logic which deals with a single-sorted domain of discourse and whose quantifiers do not range over sets, relations or predicates.
[35] A relation is an operator which assigns elements in the domain of discourse to a truth value.
[36] Functions are those operators which assign elements in the domain of discourse to other elements in the domain of discourse.
[37] These variables range over the domain of discourse.
[38] We define $\forall$ as per the following : $(\forall x\, A :_{df} \neg\exists x \neg A)$
[39] We here use the axiom schemata outlined in (Sakharov).
[40] (Ibid.)

[*] We follow standard convention regarding parenthetical dropping, wff and term formation, quotation and uniform substitution.

We will now lay the framework for how to imbue our formal calculus L_2 with a *semantics* thereby giving our 'lifeless' symbols meaning. To accomplish this, we turn to model theory. Briefly, model theory was initiated to aid in the investigation of formal languages. Model theory has been extended into a full-blown mathematical enterprise which seeks to "study the interpretation of any language, formal or natural, by means of set-theoretic structures."[41] This enables the investigation of new symbols that are added to a formal system and reveals how different formal systems, or sentences in different formal systems, can be related or meaningfully derived from each other. Model theory employs a relevant notion of structure for both scientific and mathematical structuralism. Briefly, each mathematical *structure* is tied to a particular first-order language.[42] We spell this out more formally[43]:

[2.36] A first-order *structure* K is defined as an ordered pair $\{|K|, \Phi\}$.

[2.37] $|K|$ is the *universe* or *domain of discourse* for the *structure* K.

[2.38] Φ is a function whose domain is a set of non-logical symbols.
[2.39] This domain is called the *signature* of K, such that:
[2.40] $\text{sig}(K) = \{R_1, R_2, R_3, \ldots, R_m, f_1, f_2, f_3, \ldots, f_z, c_1, c_2, c_3, \ldots, c_w\}$
[2.41] To each *n*-ary relation symbol $R \in \text{sig}(K)$ we assume that Φ assigns an *n*-ary relation: $R \subseteq |K|^n$
[2.42] To each *n*-ary *function* symbol $f \in \text{sig}(K)$ we assume that Φ assigns an *n*-ary function, such that $f : |K|^n \rightarrow |K|$
[2.43] To each constant symbol $c \in \text{sig}(K)$ we assume that Φ assigns an individual constant: $c \in |K|$

[2.44] Given a structure K and a *sentence* σ such that $\text{sig}(\sigma) \subseteq \text{sig}(K)$ we write '$K \models \sigma$' or K *satisfies* σ (σ is *true* in K).
[2.45] Let S be a set of sentences. A *model* of S is a structure M such that $M \models \sigma$ for all $\sigma \in S$, and $\text{sig}(M) = \text{sig}(S)$.
[2.46] The *class of all models* of S is denoted $\text{Mod}(S)$. A sentence τ is said to be a *logical consequence* of S (written $S \models \tau$) if $\text{sig}(\tau) \subseteq \text{sig}(S)$, and $M \models \tau$ for all $M \in \text{Mod}(S)$.
[2.47] A *theory* is a set T of sentences which is consistent and closed under logical consequence; in other words, T has at least one model, and $\tau \in T$ whenever τ is a sentence such that $\text{sig}(\tau) \subseteq \text{sig}(T)$ and $M \models \tau$ for all $M \in \text{Mod}(S)$.
[2.48] If A is a model class, we write $\text{Th}(A)$ for the *theory of A*, i.e. the set of sentences σ such that $\text{sig}(\sigma) \subseteq \text{sig}(A)$ and $M \models \sigma$ for all $M \in A$.
[2.49] If T is a theory and $S \subseteq T$, we say that S is a *set of axioms* for T if

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[41] (Hodges)
[42] We will here investigate only the first-order model theory.
[43] definitions from (Simpson, 9-11)

T = Th(Mod(S)). If there exists a finite set of axioms for T, we say that T is *finitely axiomatizable*.

[2.50] Two structures K and B are said to be *isomorphic* (written $K \cong B$) if sig(K) = sig(B) and there exists an isomorphic map of K onto B, i.e. $i : |K| \to |B|$ such that:
[2.51] i is one-one and onto;
[2.52] $R^K(a_1, \ldots, a_n)$ if and only if $R^B(a_1, \ldots, a_n)$;
[2.53] $i(f^K(a_1, \ldots, a_n)) = f^B(i(a_1), \ldots, i(a_n))$;
[2.54] $i(c^K) = c^B$.

We see then that a structure, in the model-theoretic or Tarskian sense, is a way of linking a formal calculus (such as L_2) with a semantics:

> "A structure contains interpretations of certain predicate, function and constant symbols; each predicate or function symbol has a fixed arity. The collection K of these symbols is called the *signature* of the structure. Symbols in the signature are often called *nonlogical constants*, and an older name for them is *primitives*. The first-order language of signature K is the first-order language built up using the symbols in K, together with the equality sign =, to build up its atomic formulas. … If K is a signature, S is a sentence of the language of signature K and *A* is a structure whose signature is K, then because the symbols match up, we know that *A* makes S either true or false."[44]

A relvent notion here is the concept of a *signature*. The signature of a language differentiates it from a (purely) logical calculus and/or from another formal language (hence the employment of the word 'signature') and is often employed as a shorthand to refer to particular langauges: "A *language L* is a set consisting of all the logical symbols with perhaps some constant, function and/or relational symbols included… Note that all the formulas of *L* are uniquely described by listing only the constant, function and relation symbols of *L*."[45]

We now turn to the notion of models. For example we say that "$R = \{\mathbf{R}, <, +, \cdot, 0, 1\}$ and $Q = \{\mathbf{Q}, <, +, \cdot, 0, 1\}$, where **R** is the reals, **Q** the rationals, are models for the language $L = \{ <, +, \cdot, 0, 1\}$."[46] As mentioned in [2.44] above: Given a structure K and a *sentence* σ such that sig(σ) $\subseteq$ sig(K) we write '$K \models \sigma$'. We can then via [2.45] say: A *model* of S is a structure M such that $M \models \sigma$ for all $\sigma \in S$, and sig(M) = sig(S). We then denote 'the *class of all models* of S' with 'Mod(S)', and can specify a particular model of S: 'W, $W \in$ Mod(S)'.

3. The Semantic View of Scientific Theories

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[44] (Hodges)
[45] (Weiss and D'Mello, 5)
[46] (Weiss and D'Mello, 13)

The older ‘syntactic’ or ‘recevied view’ of scientific theories was developed by philosophers motivated toward a logical empiricism.[47] This received view holds that “a model is a *model for a theory* and thus just another interpretation of the theory's calculus”[48] and “that a theory is an uninterpreted, or partially interpreted, axiom system plus correspondence rules, or co-ordinating definitions, that mediate so as to provide for the theory-world connection.”[49]

Various logical empiricist philosophers advanced this view upon recognizing “that some elements of our theoretical knowledge seem to be independent of the empirical facts.”[50] For example, “Newton’s second laws states that the force on a body is proportional to the rate of change of its momentum, where the constant of proportionality is part of what gives meaning to the concepts employed to describe the phenomena.”[51] The logical empiricists argued that:

> “a physical theory can be split into a part that expresses the definitions of the basic concepts and the relations among them, and a part which relates to the world. The former part also includes the purely mathematical axioms of the theory, and trivially all the logical truths expressible in the language of theory. If this part of the theory constitutes a priori knowledge, it is purely of matters of convention. The factual content of the theory is confined to the latter part, so the fundamental empiricist principle that the physical world cannot be known by pure reason is satisfied.”[52]

Its opposite, the semantic view prevails in framing the contemporary scientific structuralism.[53] This position “rejects the need for, and possibility of, correspondence rules and instead uses models, in the Tarskian sense, to provide an unmediated theory-world connection.”[54] This view identifies a scientific theory with a particular set-theoretic or class of model-theoretic structures[55] as per section two: “That is, the semantic view ‘construes theories as what their formulations refer to when the formulations are given a (formal) *semantic* interpretation’… Theories on this view are not linguistic, but rather abstract, set-theoretic entities – models of their linguistic formulations.”[56]

Perhaps the most glaring problem facing the syntactic view is its reliance on a distinction between theoretical and observational terms.[57] This is controversial notion that has met

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[47] (Ladyman and Ross, 111)
[48] their emphasis; (da Costa and French [1], S117)
[49] (Brading and Elaine Landry [1], 6)
[50] (Ladyman and Ross, 111)
[51] (Ibid.)
[52] (Ladyman and Ross, 111-2)
[53] (Brading and Landry [1], 5)
[54] (Brading and Landry [1], 7)
[55] detailed exploration of ‘set-theoretic predicates’ and model-theoretic model classes will follow further below
[56] (Chakravartty [1], 325-6)
[57] (Ladyman and Ross, 115)

with great criticism: "It has been widely argued (for example by Punam 1962) that there is no objective line to be drawn between observational and theoretical language, and that all language is theory-dependent to a degree."[58] I note also that the syntactic view's reliance on linguistic formulations generates a host of problems due to its need for rules of correspondence in order to secure the referents and truth-conditions for those linguistically formulated theories. *Prima facie* these considerations suggest to me that an account of scientific theories, and how the putative truth of these theories, which is not 'language' dependent (in this way) is much more preferable as a means to either secure the truth, or observing the actual practice, of the contemporary scientific activity.

I will not go into great detail regarding the many other problems levied against the syntactic view here[59] – for our purposes we will agree with Ladyman and Ross that the preferred way to understand, interpret and analyze scientific theories is "in terms of mathematical and formal structures"[60] - that we have reason to prefer the semantic view, despite its many problems[61], over the syntactic view.

However, there is signaficant disagreement over just how we are to understood the semantic view and the notion of 'model' that it espouses: "despite all the discussion about the concept of `model' of a physical theory in the literature, the precise characterization of this concept remains elusive." [62]

> "For some a model, as well as being a 'structure' that satisfies certain axioms, also includes a mapping from elements of a linguistic formulation to elements of that structure… Others hold that one should not think of a model as *including* any such interpretation of sentences... The models at issue are by definition simply those that satisfy, for example, the mathematical equations of a quantitative theory, such equations being linguistic devices; but theories themselves are models in the sense of "pure" structure: abstract entities and relations among them, excluding the linguistic formulations with which they may be linked."[63]

Despite, perhaps lacking a specific definition of model, it is clear that model-theoretic structures and models (as per section two) motivate the semantic view. McEwan observes that proponents of this view tend to assert that "a theory… is a collection of model-theoretic models…" and "to present a theory, we define the class of its models directly."[64]

Some commentators have alleged that this characterization of the semantic view is 'naïve' and ought to be abandoned. Unfortunately, "the way the semantic view is often

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[58] (Ibid.)
[59] see (Ladyman and Ross, 111-8) for a more thorough list of objections against the syntactic view
[60] (Ladyman and Ross, 118)
[61] see (Chakravartty [1]) and (Krause and Bueno) for recent objections to the semantic view; see (Suárez [2]) for criticism of the isomorphic representation view.
[62] (Krause and Bueno, 4)
[63] (Chakravartty [1], 326)
[64] (McEwan, 2)

presented, there is little said by its proponents to distance themselves from the naïve view"[65] and "the semantic view, might easily be conflated with the naïve view"[66] and a number of distinct problems arise if we do conflate the semantic view with the naïve view. McEwan has argued that the naïve view would lead us to conclude that "models are both truth-bearers and truth-makers"[67] and that "when defining [a] theory we must independently identify all the instances when the theory is true" thereby rendering the truth of our theory, for any model, trivial.[68] "These problems undermine the naïve view as a plausible account of scientific theories"[69]

Two other noteworthy problems arise from the way that the semantic view has traditionally been framed in. The problem of representation looms large:

> "The first and, arguably, the most serious problem confronting the realist or empiricist who adopts the naïve view is the *problem of representation*: neither theories nor models on the naïve view play a representational role. This problem is a result of aspects … of the naïve view. A theory is identified with a class of models and the truth of a theory is wholly determined by class membership. In this analysis the only things considered are model-theoretic models and classes of model-theoretic models. Where then does `the world' come into play?"[70]

there are meta-mathematical considerations as well:

> "Model theory, which has been the inspiration for much that has been said on models of scientific theories in general, articulates the notion of a model for *formal* first-order axiomatic systems only… But scientific theories, in general, are described only informally (consider, for instance, the theories in biology),and involve more than first-order languages. As a result, we don't have a corresponding well defined "model theory" in such cases."[71]

While these problems remain undecided, a few strategies have been attempted by both realists and empiricists alike in order to address some of these concerns; particularly regarding the problem of representation – i.e. just how theories and models, under the semantic view, 'link to the world'. Defenders of the semantic view have appealed to a notion of 'pragmatic truth' "which holds that the 'pragmatic' truth of an assertion depends on its practical effects or consequences, formulated in terms of a set of basic propositions, the latter being accepted as true in the correspondence sense"[72], set-theoretic predicates and partial structures as a

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[65] (Ibid.)
[66] (McEwan, 7)
[67] (McEwan, 4)
[68] (McEwan, 5)
[69] (McEwan, 7)
[70] (McEwan, 4)
[71] (Krause and Bueno, 4)
[72] some references removed, see original; (da Costa and French [2], 254-5)

way to meet some of these challenges. I will here attempt to flesh out those latter two ideas.

Set-theoretic predicates and partial structures have been proposed as ways to (perhaps) resolve some of the ambiguity surrounding just what models are and to give a better account of scientific theory revision. Ostensibly, these tools also provide a link between data models and the sometimes Byzantine mathematical edifice employed in scientific theory.

In order to give a more precise notion of just what a scientific theory amounts to under the semantic view and to clarify the positive claim being made by those who are proponents of the semantic view, a number of commentators have been lead to the position that 'a scientific theory is to be identified with, or given by, a particular set-theoretic predicate.' Recall that if "a model-theoretic model is an interpretation on which some set of sentences are true, then one normally takes a theory to be a set of sentences expressed in some formal language. We can call this the model-theoretic notion of a theory, or *model-theoretic theory*. A model-theoretic theory is true for a given model if and only if each of the theory's sentences are true in that model. Associated with every model-theoretic theory is a class of model-theoretic models, where the theory is true for each model which comprises this class…"[73]

This leads us to "call the models found in Tarski's analysis and the models of model-theory *model-theoretic models*" remembering that "a model-theoretic model is an interpretation which satisfies some set of sentences (or sentential formulae). An interpretation specifies a set of individuals (the domain or universe of discourse) and defines of all the appropriate symbols (i.e., constant, function and predicate symbols) of the language on that set." [74]

Following the conventions laid down in section two - that we can denote 'the *class of all models* of *S*' with 'Mod(*S*)', or can specify a particular model of *S*: 'W, $W \in$ Mod(S)' – and that the semantic view has traditionally advocated that "Theories… are not linguistic, but rather abstract, set-theoretic entities – models of their linguistic formulations"[75], it follows that the semantic view sees a scientific theory *S* (i.e. a set of propositions or sentences in a language, axiomatic or not) as being given by a class of models *W*. It is then possible to stipulate a defining characteristic of all W, $W \in$ Mod(S).

With that in mind, it has been argued: "to axiomatize a theory is to define a set-theoretical predicate"[76]:

> "da Costa and Chuaqui (1988) have convincingly argued that these set-theoretical predicates can be identified with the Bourbaki species of structures. A scientific theory may thus be characterized by a set-theoretic

[73] (McEwan, 4)
[74] (Ibid.)
[75] (Chakravartty [1], 325-6)
[76] inner quotation removed, see original; (da Costa and French [2], 253)

> predicate in such a way as to connect this approach with standard (mathematical) model theory."[77]

Thus, for example, "'classical particle mechanics' (that is, the models of a classical particle mechanics) emerges from structures of the form $\langle P, s^{\rightarrow}, m, f^{\rightarrow}, g^{\rightarrow} \rangle$, where P is the set of 'particles', $s^{\rightarrow}$ is the position function, m is the mass function, $f^{\rightarrow}$ stands for the internal forces and $g^{\rightarrow}$ represents the external force function - all of them obeying certain postulates."[78]

This approach is motivated from the fact that one can develop set-theoretic predicates to describe different mathematical structures.   A group, for example, can be considered as any n-tuple (signature) which satisfies the following conditions[79]:

$\text{Џ}_G = \langle A, \circ, *, I \rangle$

where A is a non-empty set, $\circ$ is a binary operator on A, $*$ a unary operator on A, and I and element of A, such that:

1. $(x \circ y) \circ z = x \circ (y \circ z)$
2. $x \circ I = I \circ x = x$
3. $x \circ x^* = x^* \circ x = I$

The corresponding predicate is then:

> $P(x) \leftrightarrow \exists A \exists B \exists C \exists D$ (x = $\langle A,B,C,D \rangle \wedge A$ is a non-empty set $\wedge$ B is a binary operation on $A \wedge C$ is unary operator on $A \wedge D$ is an element $A \wedge \forall x \forall y \forall z$ $((x,y,z) \in A \rightarrow (xBy)Bz = xB(yBz)$ etc.))

"The structures which satisfy this predicate are the models of the theory. That is, when a theory is fornalized in this way, the mathematical structures which satisfy the predicate are the models of this predicate, or the structures of this species of structure, P(x), or more simply, P."[80]

I will now turn to the second important strategy, which is currently being pursued as part of the programme for Ontic Structuralism, is found in the development of partial structures. The scientific structuralist capitalizes on the semantic view of scientific theories. By identifying scientific theories with model-theoretic structures, the scientific structuralist is presumably able to meet the challenges of the notorious pessimistic meta-induction. We will now demonstrate how this challenge might be met. The gist of this idea is that a scientific model might be extended across, or embedded into, another scientific model[81]:

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[77] (da Costa and French [2], 253)
[78] some notation altered, see original; (Krause and Bueno, 3)
[79] work borrowed from (da Costa and French [2], 253)
[80] (da Costa and French [2], 253-4)
[81] We borrow our definitions and work from (Bueno, 225-231) and (French, 105-6).

Essentially, the structures that are talked about under the partial structures programme are variations on the model-theoretic notion of structure such that:

[3.0] $a = \langle A, R_i \rangle_{i \in I}$

where A is the set of individuals of the domain under consideration and R_i, $i \in I$, is a family of partial relations[82] defined on A. If A is a non-empty set, then an n-place *partial relation R* over A is a triple $\langle R_1, R_2, R_3 \rangle$, where R_1, R_2, and R_3 are mutually disjoint sets, with $R_1 \cup R_2 \cup R_3 = A^n$, and such that:

[3.1] R_1 is the set of n-tuples that belong to R.
[3.2] R_2 is the set of n-tuples that do not belong to R.
[3.3] R_3 is the set of n-tuples for which it is not defined whether they belong or not to R.

A partial structure 'a' can then be extended into a total structure via so-called a-normal structures, where the structure $b = \langle A', R_i' \rangle_{i \in I}$ is said to be an a-*normal structure* if:

[3.4] $A = A'$
[3.5] Every constant of the language in question is interpreted by same object both in a and in b, and
[3.6] R_i' extends the corresponding relation R_i, in the sense that, each R_i' is defined for every n-tuple of objects of its domain; such that for every $x \subseteq A^n$, x falls under either R_1 or R_2.

A sentence 's' is then said to be quasi-true (in 'a' according to 'b') if:

[3.7] 'a' is a partial structure.
[3.8] 'b' is an a-normal structure; and
[3.9] 's' is true in 'b' in conformity with Tarski's definition of truth.

Formally, if we have two partial structures:

$$a = \langle A, R_i \rangle_{i \in I} \qquad \text{and} \qquad a' = \langle A', R_i' \rangle_{i \in I}$$

(where R_i and R_i' are partial relation as above, so that $R_i = \langle R_1, R_2, R_3 \rangle$ and $R_i' = \langle R_1', R_2', R_3' \rangle$) then a function f from A to A' is a *partial isomorphism* between a and a' if:

[3.10] f is bijective and
[3.11] for all $x, y \in A$, $R_1xy \leftrightarrow R_1'f(x)f(y)$ and
[3.12] for all $x, y \in A$, $R_2xy \leftrightarrow R_2'f(x)f(y)$

[82] "[For] the sake of simplicity, [I] omit operations and take structures to be a domain endowed with certain relations. This is can be done without loss of generality because operations reduce to relations" and "Basically, an operation taking *n* arguments is equivalent to a *n*+1 place relation." (Frigg, 55)

Note: If $R_3 = R'_3 = \varnothing$, we no longer have *partial structure* but 'total' ones which recovers the standard notion of isomorphism.

"Thus, two models may be related not by inclusion but by this somewhat weaker notion of partial isomorphism which captures the idea that they may share parts of their structure."[83] As a note, "*Partial homomorphism* is obtained if injection replaces bijection in condition [1] immediately above and implication replaces equivalence in condition [2] immediately above."[84]

Using the partial structures approach it has been shown that a sort of hierarchy where data models occupy the lowermost position can be linked to the abstract mathematical models which occupy the highest position. "Thus, in terms of the hierarchy given in [The Application of Bose-Einstein Statistics] … and putting things a little crudely, we have something like the following:

$A_4 = <D_4, R_{4i}, f_{4j}, a_{4k}>_{i\in I,\, j\in J,\, k\in K}$
$A_3 = <D_3, R_{3i}, f_{3j}, a_{3k}>_{i\in I,\, j\in J,\, k\in K}$
$A_2 = <D_2, R_{2i}, f_{2j}, a_{2k}>_{i\in I,\, j\in J,\, k\in K}$
$A_1 = <D_1, R_{1i}, f_{1j}, a_{1k}>_{i\in I,\, j\in J,\, k\in K}$

where A_4 represents the mathematical structure of group theory (and where Redhead's surplus structure is represented in terms of an associated family of structures a_{4k}, $k\in K$), A_3 represents the theory of Bose-Einstein statistics, A_2 represents London's model, with all its idealizations and A_1, for simplicity, is a condensed representation of the data models, experimental models and so forth."[85]

4. Additional Concerns

Due to space constraints, a full survey of the ongoing debate surrounding both just how we are to understood the semantic view and the plausibility of the semantic view cannot be carried out here. However, as noted in section three the *problem of representation* looms large – asking us how a scientific theory, under the semantic view, can be understood to represent something in the world? Additionally, some commentators have alleged that *isomorphism* is not an appropriate way to establish representation. They have asserted the rather evident observation that "Isomorphism is symmetric, reflexive and transitive while representation is not."[86] While weaker relations have been offered to replace isomorphism including homomorphism, partial isomorphism (above) and Swoyer's Δ/Ψ-morphism,[87] all of them run into similar problems.

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[83] (French, 106)
[84] (French, 106-7)
[85] (French, Bueno and Ladyman, 514)
[86] (Frigg, 10)
[87] (Suárez, 35,38)

When we say that "Isomorphism is symmetric, reflexive and transitive while representation is not" exactly what are we saying that representation is not? Now, presumably if 'x represents y' then x stands in relation to y.

[4.0] A relation R on a set B is called *reflexive* if and only if $<a, a> \in R$ for every $a \in B$.

[4.1] A relation R on a set B is called *irreflexive* if and only if $<a, a> \notin R$ for every $a \in B$.

[4.2] A relation R on a set B is called *symmetric* if and only if whenever $<a, b> \in R$ it is also the case that $<b, a> \in R$ for any $a, b \in B$.

[4.3] A relation R on a set B is called *antisymmetric* if and only if whenever $<a, b> \in R$ and $a \neq b$ it is the case that $<b, a> \notin R$ for any $a, b \in B$.

[4.4] A relation R on a set B is called *transitive* if and only if whenever $<a, b> \in R$ and $<b, c> \in R$ it is the case that $<a, c> \in R$ for any $a, b, c \in B$.

Surely something can represent itself: "I don't say that a representation is something that represents something *else*, because a representation can represent itself. (To take a philosophically famous example, the 'Liar Paradox' sentence 'This sentence is false' represents the quoted sentence itself)."[88] But this does not necessarily imply that everything represents itself. Thus representation is not necessarily *reflexive* nor either *irreflexive* failing conditions [4.0] and [4.1]. Similarly, for some x that represents some y it might be the case that y also represents x. But, this is not sufficient to suggest that representation is *symmetric* nor either *antisymmetric* failing conditions [4.2] and [4.3]. Isomorphism, then, appears to be 'too strong' to serve as our vehicle for representation.

One area that has been unfortunately neglected (and this has been a point raised by a number of philosophers[89]) in most philosophical discourse about models is that important area where models and theories are linked to data (or vice-versa). To overcome this Brading and Landry have suggested that scientific theories, under the semantic view, should not be construed as *representing* but rather as *presenting*. They argue for "a minimal construal of both scientific structuralism and structural realism"[90] which they suggest is "a *methodological stance*" whereby "we forgo talk of 'the structure of the phenomena' and simply begin with data models"[91] in order to side-step the *problem of representation* all-together.

Recall that as per section three, a scientific theory was characterized as a class of models taking the form:

"$A_4 = <D_4, R_{4i}, f_{4j}, a_{4k}>_{i \in I, j \in J, k \in K}$
$A_3 = <D_3, R_{3i}, f_{3j}, a_{3k}>_{i \in I, j \in J, k \in K}$
$A_2 = <D_2, R_{2i}, f_{2j}, a_{2k}>_{i \in I, j \in J, k \in K}$

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[88] (Crane, 10)
[89] see (Cartwright)
[90] (Brading and Landry [1], 26)
[91] (Brading and Landry [1], 9)

$A_1 = \langle D_1, R_{1i}, f_{1j}, a_{1k} \rangle_{i \in I,\, j \in J,\, k \in K}$”[92]

where A_1 is the class of data models for that particular scientific theory.

For Brading and Landry, the move from *presentation* to *representation* hinges on “connecting data models to the phenomena.”[93] “Connecting theoretical models to data models… can be accounted for solely in terms of *presentation* of shared structure.” However, connecting data models to the phenomena “demands the addition of something more.”[94] “Thus, two things are required to connect the high level theory to the phenomena: an experimental *theory of the data* and an empirical *theory of the phenomena*.”[95]

Hence, for Brading and Landry, the move to ontological realism requires a theory of representation, which in turn requires a theory linking the structure of data to the structure of phenomena. “From an *empirical stance*, one may hold that what structures the phenomena is the high-level theory, whereas from a *realist stance* one may hold that what structures the phenomena is the world.”[96] Thus, “van Fraassen, as a ‘structural empiricist’, suggests that we simply *identify* the phenomena with the data models.”[97] Consider:

> “An empirical model is one which is derived from and based entirely on data. In such a model, relationships between variables are derived by looking at the available data on the variables and selecting a mathematical form which is a compromise between accuracy of fit and simplicity of mathematics… The important distinction is that empirical models are not derived from assumptions concerning the relationships between variables, and they are not based on physical laws or principles. Quite often, empirical models are used as ‘submodels,’ or parts of a more complicated model. When we have no principles to guide us and no obvious assumptions suggest themselves, we may (with justification) turn to data to find how some of our variables are related.”[98]

This does not sit very well with them for “if we are to be motivated to move beyond the more modest methodological stance we need reasons. In particular, if we are to adopt either the empiricist or the realist alternative, we need a justification for the claim that data models share the same structure as the phenomena and, as a result, that the former can be taken as representations of the latter.”[99]

However, “from a *realist* stance we may say that what structures the phenomena is the world” such that “Structural realists, such as French and Ladyman, who adopt a realist stance and so presume that the world structures the phenomena, invoke the ‘no miracles’ argument to explain the necessity of identifying the structure of data models and the structure of the

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[92] (French, Bueno and Ladyman, 514)
[93] (Brading and Landry [1], 18)
[94] (Ibid.)
[95] (Brading and Landry [1], 8)
[96] (Brading and Landry [1], 21)
[97] (Brading and Landry [1], 26)
[98] (Edwards and Hamson, 102)
[99] (Brading and Landry [1], 21)

phenomena; it is used to argue that if there was no shared structure between the (data models of the) theory and the world (the phenomena) the success of science would be a miracle."[100]

This is the path I would pursue as a scientific realist. Essentially, to explain the adequacy and application of mathematical models and structures to the empirical without the world, in some sense, being inherently structured would require a miracle. While just what structure the phenomena takes and just what structure our data takes is subject to some revision, refinement and improvement – the no miracles argument requires that the structure of the data (approximately) represents the structure of the phenomena.

While this response may provide a plausible out for the proponent of the semantic view in the face of the representation problem, it does not address the shortcomings of the Isomorphic Theory of Representation.

5. Scientific Structuralism

Worrall introduced Epistemic Structural Realism (ESR) as a way to preserve realism in scientific truth without committing to a full-fledged traditional scientific realism. There are two primary arguments compelling ESR's inception: the *no-miracles* argument and the *pessimistic meta-induction* argument. The *no-miracles* argument is an abductive inference[101] which takes the general form:

"(1) *p*
(2) *q* is the best explanation of *p*
∴ *q*"[102]

Such that

"(1) Mature scientific theories are empirically successful.
(2) The (approximate) truth of mature scientific theories is the best explanation of their empirical success.
∴ Mature scientific theories are (approximately) true."[103]

The *no-miracles* argument asks us how the predictive powers of scientific theories are so successful if science is not really telling us about real things in the world (which, the scientific realist argues, would require a miracle).

The *pessimistic meta-induction* is essentially the claim that "we cannot commit ourselves to the belief in present theories since successful theories throughout the history of science were refuted or abandoned."[104] Traditionally, scientific realism asks us to accept that the

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[100] (Brading and Landry [1], 23)
[101] many commentators have noted that the question of realism in the philosophy of science may really come down to whether one accepts the legitimacy of 'reasoning to the best conclusion' (i.e. abduction) or not
[102] (Wüthrich, 13)
[103] (Ibid.)
[104] (Kantorovich, 2)

objects talked about by scientific theories are real, by this antirealist argument it is asserted that we cannot take scientific theories or their ontologies at face-value because they will likely be discarded.

Worrall advanced ESR as a way to achieve a middle-ground between these two positions. "ESR purports to identify the structural content of a theory in such a way as to ensure cumulative continuity in that kind of content."[105] Hence, ESR is concerned with the preservation of scientific continuity which has been disputed by such thinkers as Kuhn[106] and the *pessimistic meta-induction*. Scientific structuralism responds to the charges of discontinuity by asserting that certain structural features of differing scientific theories remain stable, even in the face of radical changes to the scientific ontology throughout history,[107] providing a point upon which to affirm the *no-miracles* argument (science is the accumulation of structural information). Essentially, ESR asks us to commit only to the mathematical content of scientific theories and admits that our actual knowledge of *things-in-themselves* is limited at best.

To illustrate this I will now turn to a more general view of structure:

> "Here, for simplicity, we shall take structures of the form $K = \langle A, R \rangle$, where A is a non empty set and R is a binary relation on A, … in doing physics one needs more than first order structures, but the considerations to be made here can be seen as a first sketch of an idea which can be generalized for higher order structures…"[108]

Let $R = \{(a, b), (a, c), (c, d), (d, b)\}$. We can now derive the following simple diagram:

$$\begin{matrix} a & \rightarrow & b \\ \downarrow & & \uparrow \\ c & \rightarrow & d \end{matrix}$$

Figure 1: *R*

> "According to Russell, the 'structure' of *R* should not depend on the particular terms forming part of the field of the relation, which should be modified without altering its structure."[109]

Following Russell's notion of structure we can use Figure 1 to derive:

$$\begin{matrix} \cdot & \rightarrow & \cdot \\ \downarrow & & \uparrow \\ \cdot & \rightarrow & \cdot \end{matrix}$$

Figure 2: The structure of *R*.

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105 (Saatsi, 2)
106 see (Kuhn)
107 (Brading and Landry [2], 21)
108 (Krause [2], 115)
109 (Krause [2], 116)

Structuralists, in their respective areas of study, tend to assert that these extracted structures are what we can investigate and have knowledge of. As Russell states about the physical:

> "We can know *only* what is required in order to secure the correspondence. That is to say, we can know nothing of what it is like in itself, but we can know the sort of arrangement of physical objects which results from their spatial relations… Thus we come to know much more about the *relations* of distances in physical space than about the distances themselves."[110]

Thus, according to Russell, what we cannot know is physical things as they are *in-themselves* but what we can know are the abstract relations by which those *things-in-themselves* are arranged. On this view, what we perceive is in correspondence with what *is*.

Ontic Structural Realism (OR) is a more radical thesis. Two of the most predominant advocates of this position, French and Ladyman, state it thus:

> "We regard the ontic form of SR as offering a reconceptualisation of ontology, at the most basic metaphysical level, which effects a shift from objects to structures. Now, in what terms does such a reconceptualisation proceed? This hinges on our prior understanding of the notion of an 'object' which has to do, as we have indicated, with the metaphysics of individuality."[111]

The traditional view of object-hood is grounded in the notions of *quiddity*, or the notion that each thing has its own essence, and *haecceity*, or the properties and attributes which make something its own particular thing.[112] It should be noted that a physical entity has been traditionally defined as a three-dimensional individual object located in spacetime.[113]

OR denies the epistemic limitations of ESR by asserting a revisionist metaphysical claim: that our traditional ontological category of object-hood is misguided[114] that "Structures have ontological primacy over objects' and this either means [1] that structures are all that exist or [2] that entities are dependent for their own existence on the existence of

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[110] his emphasis; (Russell, 14)

[111] (French and Ladyman [1], 37)

[112] these terns are somewhat interchangeable though the prior bears the connotation *what it is* and the latter holds the connotation *which it is*

[113] (Heller, 4); However, there is still much debate over this definition. Those who hold that physical objects exist only spatially in three dimensions, such that they exist *in their entirety* at every moment in time, are called Endurantists. Those who hold that a physical object exists spatio-temporally in four dimensions, such that an object is identified *as the sum of its temporal parts*, are called Perdurantists; (Dainton, 39-40)

[114](Chakravartty, 867-868)

structures."[115] Thus, OR may do away with *quiddity* or *haecceity* altogether. Such a move is significantly motivated by work in quantum mechanics[116] where the status of individuality and object-hood are underdetermined.[117] For example, Leibniz's Principle of the Identity of Indiscernibles (PII)[118] appears to fail for many subatomic particles.

What does this mean? First, that physical entities may exist *in virtue of* structure and second, that the traditional characterization of abstract versus physical objects is likely to narrow. As Shapiro puts it "the structuralist perspective is a healthy blurring of the distinction between mathematical and ordinary objects."[119] Resnik, a mathematical structuralist, explains this further:

"Let me start by considering some physical objects that appear to be as much mathematical as physical. I have in mind quantum particles. The Ontic Structural Realism (OR) is a more radical thesis. Two of the most predominant advocates of this position, French and Ladyman, state it thus:

> "We regard the ontic form of SR as offering a reconceptualisation of ontology, at the most basic metaphysical level, which effects a shift from objects to structures. Now, in what terms does such a reconceptualisation proceed? This hinges on our prior understanding of the notion of an 'object' which has to do, as we have indicated, with the metaphysics of individuality."[120]

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[115] some inner quotations removed for clarification, please consult original; (Kantorovich, 17-18)
[116] see (Rickle and French)
[117] (Krause, 162)
[118] $\forall F(Fx \leftrightarrow Fy) \rightarrow x=y$; this essentially says two things are identical when all the properties that are true of one thing are the same as all the properties that are true of the other (and vice-versa)
[119] (Shapiro, 256)
[120] (French and Ladyman [1], 37)
[121] these terns are somewhat interchangeable though the prior bears the connotation *what it is* and the latter holds the connotation *which it is*
[122] (Heller, 4); However, there is still much debate over this definition. Those who hold that physical objects exist only spatially in three dimensions, such that they exist *in their entirety* at every moment in time, are called Endurantists. Those who hold that a physical object exists spatio-temporally in four dimensions, such that an object is identified *as the sum of its temporal parts*, are called Perdurantists; (Dainton, 39-40)
[123](Chakravartty, 867-868)

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> "Let me start by considering some physical objects that appear to be as much mathematical as physical. I have in mind quantum particles. The term 'particle' brings to mind the image of a tiny object located in space-time. But, on what seems to be the consensus view of the puzzling entities of quantum physics, this image will not do. Most quantum particles do not have definite locations, masses, velocities, spin, or other physical properties most of the time."[129]

He elaborates:

> "We began by observing that it is usual for philosophers of mathematics to distinguish between supposedly abstract, mathematical objects and supposedly concrete, physical ones by appealing to the spatio-temporal locatability, causal powers, or detectability of the latter. We now see that distinguishing between abstract and concrete objects in this way is obsolete."[130]

It should be noted that this particular problem, along with the ontological status of the wave function, has served as a traditional point of division between realists and antirealists in the philosophy of science. If one adopts scientific realism, that being realism in both scientific truth and ontology, and takes contemporary quantum physics (with its mathematical edifice and strange ontology) at face-value it appears that the traditional category of object-hood and the traditional divisions between physical/abstract or physical/mathematical have already been narrowed, regardless of the metaphysical stance of OS. Let us now turn to the mathematical structuralism.

6. Mathematical Structuralism

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[124] some inner quotations removed for clarification, please consult original; (Kantorovich, 17-18)
[125] see (Rickle and French)
[126] (Krause, 162)
[127] $\forall F(Fx \leftrightarrow Fy) \rightarrow x=y$; this essentially says two things are identical when all the properties that are true of one thing are the same as all the properties that are true of the other (and vice-versa)
[128] (Shapiro, 256)
[129] (Resnik, 102)
[130] (Resnik, 107)

Traditionally, the mathematical realist has held a Platonist conception of mathematics - that there is an abstract mind-independent mathematical reality where the actual objects of mathematical statements reside. This goes against the common view that mathematics is *all and only* the management of special symbols. Formalism was a philosophical position motivated by this commonsense view asserting that mathematics is a meaningless activity characterized by the manipulation of finite strings of symbols.[131] It was the express goal of Hilbert's Program to be able to generate a consistent set of axioms from which every possible classical mathematical theorem could be derived procedurally as a means by which to secure the absolute certainty of mathematical truth. However, this activity was more or less halted by Kurt Gödel, a Platonist, who proved with his famed Incompleteness Theorems that no such axiomatic framework was possible. The Platonist conception sees mathematics as two parts – an abstract mind-independent realm and the languages and statements which talk about that realm - although these two parts are somewhat interwoven. It should be noted that a mathematician can perform any mathematics regardless of their philosophical view of what mathematics *ought* to be, or what they believe mathematics *is*.[132]

Shapiro states that one of his primary motivations for developing *ante rem* structuralism (ARS) is to preserve this sort of conception in mathematics, or the *philosophy last* position, given the long and dignified tradition of mathematics itself.[133] However, his view does subtly contrast with traditional Platonism. The traditional view holds that numbers, much like physical things, have individual essence, *quiddity* or *haecceity*, such that 'one' is considered to be a separate entity than 'six,' what Shapiro might call the *Independence Intuition*. ARS asserts that the only essence to numbers is that they belong to the abstract structure of the natural number line.

Linnebo spells this idea out more clearly:

> "Let's now try to be more precise about what the Dependence Claim says. Consider the domain *D* of some mathematical structure. One aspect of the Dependence Claim is that in this structure *Objects Depend on Objects*:
>
> > (ODO) Each object in *D* depends on every other object in *D*.
>
> For instance, each natural number is said to depend on all the other natural numbers in a way that it does not depend on, say, sets. It follows that it is impossible for one natural number to exist without all the others existing as well. Another aspect of the Dependence Claim is that in mathematical structures *Objects Depend on Structures*:

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[131] (Maddy [2], 23-24)
[132] although this may influence their decision as to which mathematical system they will be inclined to investigate - as in the case of Intuitionist mathematics which denies the Law of Double Negation ($\neg\neg P \rightarrow P$) and the Law of the Excluded Middle ($P \vee \neg P$)
[133] (Shapiro, 3, 30)

> (ODS) Each mathematical object depends on the structure to which it belongs."[134]

One of the motivations for this position can be found in the challenge[135] that Benacerraf famously raised against set-theoretic Platonism. I will reconstruct what we hold to be the essential aspect of this argument as:

[6.0] We can define the natural number line in an infinite number of ways,
[6.1] The Zermelo Ordinals are defined $0 = \emptyset$, $1 = \{0\}$, $2 = \{1\}$, $3 = \{2\}$ and so on, such that $1 \notin 3$
[6.2] The Von Neumann Ordinals are defined as $0 = \emptyset$, $1 = \{0\}$, $2 = \{0, 1\}$, $3 = \{0, 1, 2\}$ and so on, such that $1 \in 3$
[6.3] The set-theoretic Platonist believes that the natural number line is a set, such that 1 is a set, 2 is a set and so on,
[6.4] The set-theoretic Platonist accounts for mathematical truth by saying that a mathematical statement names a set,
[6.5] In [6.1] the number one is not 'in' the number three; in [6.2] the number one is 'in' the number three, thus the relevant criterion of individuation, (PII), does not hold.[136]
[6.6] We have no reason to prefer [6.1] over [6.2] or vice-versa. How then can the set-theoretic realist say which of these formulations the natural number line *is*?

Benacerraf thus concludes that "numbers are not objects, against realism in ontology."[137]

ARS enables us to answer this question, thereby preserving a realist position. In the above case, each defined natural number system is a particular instance of an abstract natural-number structure.[138] That is to say, the two natural number systems above are isomorphic to each other and demonstrate the existence of a single abstract structure that they exemplify; that it is wrong to range (PII) over the individual numbers because there are no natural numbers as particular objects - that is, as existing things whose 'essence' or 'nature' can be individuated independently of the role they play in a structured system of a given kind.[139] Thus, our notion of identity ought to apply to the structural content of the two systems and confirms that they are identical because each 'number' in one system lines up in one-to-one correspondence with a 'number' in the second – that the relevant criterion for identity is isomorphism[140] – essentially that there exists a one-to-one "structure preserving"[141] map between two structures that preserves relations and objects held of those relations. As one can see, this distinction changes the talk of mathematics as the study of abstract objects, to mathematics as the study of abstract structures. It is

[134] (Linnebo [2], 67)
[135] (Shapiro, 5)
[136] (Brading and Landry [2], 572)
[137] (Shapiro, 5)
[138] (Shapiro, 5-6)
[139] (Brading and Landry [2], 572)
[140] (Shapiro, 93)
[141] (Shapiro, 91)

important to note that objects are retained under this ontology; however, these objects are defined by their associated relations within a structure and are ontologically dependent on structures for their existence[142] and the inner content or intrinsic properties of objects within a structure cannot be analyzed (it is mostly accepted wisdom that numbers, for example, do not have any sort of inner content or intrinsic properties). It is understood that there is usually a background ontology selected (which is understood to be structurally irreducible) or fixed to a particular structure theory[143] which Shapiro maintains is a deciding factor for adopting *ante rem* structuralism over categorical *in re* structuralism.[144]

ARS differs from the alternative mathematical structuralisms in the sense that under ARS a structure is an independent abstract universal which stands apart from the systems which may exemplify it. *Ante rem* means that "universals exist prior to and independent of any items that may instantiate them. Even if there were no red objects, the Forms of Redness would still exist."[145] The primary opposing alternative "is that universals are ontologically dependent on their instances. There is no more to redness than what all red things have in common."[146] For Shapiro, any view of universals which denies the free-standing independent *ante rem* view, he dubs *in re* universalism. Thus, *in re* structuralism (IRS) admits only structures of systems whereas the *ante rem* view admits that systems exemplify abstract free standing structures which exist regardless of whether or not a system exemplifies them.[147]

Shapiro calls a *system* "a collection of objects with certain relations"[148] between these objects giving the examples of "An extended family is a system of people with blood and marital relationships"[149] and baseball as "a collection of people with on-field spatial and 'defensive-role' relations."[150] He defines a *structure* as "the abstract form of a system, highlighting the interrelationships among the objects, and ignoring any features of them that do not affect how they relate to other objects in the system."[151] It is the task of ARS to develop a structure theory to formally model their respective positions[152] - a theory "strong enough to encompass [the behavior] of all structures."[153] A structure theory is a collection of axioms, or statements, which describe how structures behave. Shapiro outlines an axiom highly relevant to our discussion, the *Coherence Axiom*: "A structure is characterized if the axioms are coherent"[154] - If P is a coherent sentence in a second-order language, then there is a structure that satisfies (entails or "makes true") P.[155]

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[142] (Shapiro, 82)
[143] (Shapiro, 82, 86)
[144] (Shapiro, 87)
[145] (Shapiro, 84)
[146] (Ibid.)
[147] (Shapiro, 101)
[148] (Shapiro, 73)
[149] (Ibid.)
[150] (Shapiro, 74)
[151] (Ibid.)
[152] (Shapiro, 90)
[153] (Maddy [2], 173-4)
[154] (Shapiro, 133)
[155] (Shapiro, 95)

I will present one powerful arguments that have been raised against Shapiro's view:

> [*ante rem* access problem] How does the mathematician come to know structures which we do not already have systems for?[156]

This seems to me to be a compelling argument that Shapiro must account for if he is going to offer ARS as a viable philosophy of mathematics. As I have noted Shapiro's *ante rem* structuralism requires at least *three* kinds of abstract entities: *objects*, *systems* and *structures* themselves. Psillos concludes that this renders ARS as unmotivating and, thus-far, incompatible with OR:

> "One important difference between *ante rem* and *in re* structuralism concerns the role of objects in structures. Since *in re* structuralism focuses on relational systems, it takes the objects of a structure to be whatever objects systems with this structure have. According to *in re* structuralism, there are no extra objects that 'fill' the structure. It's then obvious that the objects that 'fill' the *in re* structures have more properties than those determined by their interrelationships in the structure. They are given, and acquire their identity, independently of the abstract structure they might be taken to exemplify.
>
> By hypostatizing structures, *ante rem* structuralism introduces more objects: those that 'fill' the abstract structure. Of these 'new' objects nothing is asserted than the properties they have in virtue of being places (or roles) in a structure. These places cannot be identified with the objects of any or all of the *in re* structures that are isomorphic to the abstract pattern. This is what Shapiro calls the "places-are-objects" perspective. The 'fillers' of the abstract (*ante rem*) structure are places, or positions, in the structure; yet if one considers the structure in and of itself, they are genuine objects. After all, they *must* be such since the abstract structure instantiates itself (cf. Shapiro 1997, 89). Given that an instantiated abstract structure needs objects to be instantiated into, the places of the abstract structure must be objects. Mathematical structuralism, then, does not view structures without objects. It's not revisionary of the underlying ontology of objects with properties and relations."[157]

It should be noted that while the characterization of IRS suggests nominalism, the traditional set-theoretic Platonist would probably fall under the IRS label as well (with the addition of an ontological commitment to sets). For the purposes of this paper I will consider those who embrace IRS as being those who advance an eliminative programme. I turn again to Linnebo for clarification, this time regarding the different eliminative positions: "The eliminative versions deny that there are abstract mathematical structures

[156] see (Linnebo [1], 103-4)
[157] (Psillos, 564)

and that the nature of mathematical objects is exhausted by their being positions in such structures. We can distinguish between three versions of eliminative structuralism.

> *Deductivist structuralism* avoids both ontological commitment to mathematical objects and all use of modal vocabulary. It interprets mathematics as the formulation of various (mostly categorical) theories to describe various kinds of concrete structures, and as the study of what holds in all models of each of these theories.
>
> *Modal structuralism* lifts the deductivists' ban on modal notions. It interprets mathematics as asserting that it is possible for various theories to have concrete models, and as studying what necessarily holds in all such models.
>
> Finally, *set-theoretic structuralism* rejects the deductivists' nominalism in favor of a background theory of sets. It then takes mathematics to be the study of various structures realized among the sets. This is often what mathematicians have in mind when they talk about structuralism."[158]

From the *deductivist* direction, IRS supporters have argued for the adoption of category theory as the foundational language of mathematics – to replace set theory. This view is controversial for a number of reasons:

[6.7] Category theory may 'secretly' employ/be reliant on set theory.[159]
[6.8] It isn't clear whether Category Theory eliminates talk of abstract entities.
[6.9] It has been argued that for a theory to be foundational, it must make positive ontological commitments (which would naturally be incompatible with IRS).

IRS supporters have counter-asserted that category theory serves as a framework or universal language, as opposed to being a foundation for mathematics, which organizes what we aim to say about mathematical and logical structures. In this second sense, category theory is almost diametrically opposed to ARS and its ontological commitments.

It has been suggested, following Landry, that "we may now distinguish between both *ontological realism* and *idealism* – the claim that mathematical objects (and perhaps, structures as well) exist, as either ontologically given or psychologically constructed "things", independently of their linguistic expression – from *semantic realism* – the claim that mathematical propositions, which talk about mathematical objects *qua* positions in structured systems and structured systems themselves, meaningfully and objectively refer when interpreted from *within* a linguistically presented system."[160]

Category theory has been affectionately dubbed "general abstract nonsense." This is fitting, given the extreme level of abstraction employed. Category theory is an almost entirely different way of doing mathematics (over say ZFC set theory). Essentially, a

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[158] (Linnebo [2], 61)
[159] see (Pedroso)
[160] footnotes removed; (Landry, 82)

category is a graph with a collection of nodes with arrows between those nodes, satisfying a number of conditions:

[1] A category 'C' can be described as a set 'Ob', whose members are the objects of 'C'[161], satisfying the following conditions:

[*Morphism*] For every pair 'X, Y' of objects, there is a set 'Hom(X, Y)', called the morphisms from 'X' to 'Y' in 'C'. If 'f' is a morphism from 'X' to 'Y', we write '$f: X \rightarrow Y$'.

[*Identity*] For every object 'X', there exists a morphism 'id_X' in Hom(X, X) called the *identity* on 'X'. We write '$id_X : X \rightarrow X$'.

[*Composition*] For every triple 'X', 'Y', 'Z' of objects, there exists a partial binary operation from 'Hom(X, Y) × Hom(Y, Z)' to 'Hom(X, Z)', called the composition of morphism in 'C'. If '$f: X \rightarrow Y$' and '$g : Y \rightarrow Z$', the composition of 'f' and 'g' is noted '$(g \circ f) : X \rightarrow Z$'.

[2] [*Morphism*], [*Identity*] and [*Composition*] satisfy two axioms:

[*Associativity*] If '$f: X \rightarrow Y$', '$g : Y \rightarrow Z$' and '$h : Z \rightarrow W$', then '$h \circ (g \circ f) = (h \circ g) \circ f$'.

[*Identity*] If '$f: X \rightarrow Y$', then '$(id_Y \circ f) = f$' and '$(f \circ id_X) = f$'.[162]

Immediately one can see just how radically different category theory is over the traditional ZFC. One interesting thought is how the development of category theory might impact work in (traditionally set-theoretic) model theory. But that is subject for a future discussion. Having fleshed out the two primary structuralist positions, I will note that the question over 'the proper foundations for mathematics' remains firmly unsettled. However, from the pragmatic considerations that ZFC set theory is, at the present time: (i) the accepted background framework for the overwhelming majority of mathematicians, (ii) that ZFC set theory presupposes the existence of an infinite universe of (both well-behaving and misbehaving) sets/classes and (iii) noting the out standing problems surrounding the advancement of category theory – I am lead to tacitly support the Platonist-leaning *ante rem* option.

7. Assessing the MUH

As I have demonstrated above, the various positions composing structuralism in both the philosophy of science and mathematics remain undecided and also in their embryonic stages of development. However, recent work toward an account of scientific theories, hinging on the semantic view, seems plausible. And, it would appear that if such an

[161] We make note here that Category Theory can actually be formulated *without* talk of objects.
[162] see (Marquis)

approach can succeed, then OR has a strong chance of succeeding as well via the still-compelling no-miracles argument. In the philosophy of mathematics, a Platonist position still seems the best-bet for mathematical realism. However, it is unclear as to whether or not we can effectively link the Platonist-leaning ARS option to OR. This consideration alone casts doubt on the plausibility of the MUH, at least as it has been presented here. And, when we take problems with the isomorphic theory of representation into account, it would seem to follow that Tegmark's MUH is unmotivating.

However, the option still remains that Tegmark need not commit to ARS in order to secure a Platonist-leaning mathematical structuralism. It is this basic notion that inspires the remainder of this paper.

8. A Metaphysics from Physics

The metaphysician has traditionally organized their ontology according to some sort of ontological category scheme (i.e. what kinds of entities, or groups of entities, that there are). Now while many entities have been rejected, added, reconceived or revised; there are two entities that usually find their way into ontological discussions and that have remained relatively stable throughout philosophical inquiry: *objects* and *properties*.

OR is primarily concerned with the former, as made clear above. However, consideration surrounding properties and just how properties are entertained with, or related to, objects is of interest as well. A bundle theory is the ontological claim that an object consists only of a collection (*bundle*) of properties. This contrasts directly with a *substance* approach which sees objects (and their substances) as distinctly separate from the properties which may describe them - the subsisting object may be understood in a primitive *quiddity* or *haecceity* sense (which would then be taken as separate or distinct from its properties).

There are two primary 'property theories' at the present: on the one hand we have the age-old theory of *properties as universals* (i.e. *realism*) on the other hand "Realism has a strong rival: trope ontology, according to which properties are particulars which may resemble each other."[163] A trope ontology allows us to construct classes of particular properties that resemble each other in such a way as to make sense of general names (like say 'red'). Traditionally, a universal-based ontology might see properties as instantiated in a substance such that "As instantiations they are particular – a universal is not instantiated twice at one instant in one substance. Yet because any one universal may be instantiated in two (or more) different substances any one that is will be 'wholly present' in two (or more) different regions of space or space-time; this appears to be incoherent."[164]

Many commentators have suggested that tropes and bundles seem very friendly to each other[165]; for when we combine bundle theory with *realism* significant problems seem to arise when we take (PII) into consideration as seen above. Now there is much debate over

163 (Wachter, 308)
164 (Schneider, 6)
165 See (Wayne)

these different metaphysical concepts and just how we are to understand them. Sadly, due to space constraints, I will not tread further into those considerations here. What is of present concern, is recent work aiming to link these ideas to QFT.

OR is significantly motivated by recent work in Quantum Field Theory (QFT) as "our best quantum theories are field theories" [166]. As we might expect there are many different, competing, interpretations of QFT. I will simplify these many views into the dichotomy of those that advocate a particle interpretation and those that do not. A rough definition offered by Redhead gives us a starting point from which to work: "A field theory in physics is a theory which associates certain properties with every point of space and time."[167]

To flesh this idea out a little more concretely: "As it is usually described, a field consists of values of physical quantities associated with spacetime locations or spatiotemporal relations."[168] Alternatively, "[a field is] a domain or region on which a function is defined: a field is a *field of values* of this function."[169] And, "A field is extended over all of space. There may be more than one field… A field can have different intensities (or "strengths") in different regions."[170] Thus, for example, "electromagnetic field theory associates electric and magnetic forces with space-time points."[171]

Now many commentators have either advocated a particle interpretation, or at least for the plausibility of a particle interpretation, of QFT. In a classical particle theory an individual is usually understood in a "naïve 'substance' approach."[172] "An individual is an entity which is the 'bearer' of properties – if you strip away all the properties you are left with a substratum indicated by a proper name that picks out that individual as itself at one particular time and reidentifies it 'transcendentally' as the same individual at other times in its history"[173] perhaps invoking the notions of *quiddity* or *haecceity*: "A particle theory in physics is a theory which attributes to certain individuals (the particles) a variety of properties."[174] "This definition makes it look as though space-time points are the primary individuals of the theory. The fields are then properties of these spatio-temporal individuals."[175] So it can be seen that particle theories make use of the substance approach.

On the other hand von Wachter argues: "As it is well known the concept of a permanently existent particle is not consistent with this theory [of relativity]. But rather it is the point event in space-time that is the basic concept. In principle all structures have to be understood as forms in a generalised field which is a function of all the space-time points.

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[166] (Ladyman and Ross, 138)
[167] his emphasis; (Redhead, 35)
[168] (Wayne, 16)
[169] their emphasis; (Peuquet et al., 30)
[170] (Wachter, 318)
[171] (Redhead, 35)
[172] (Redhead, 59)
[173] (Ibid.)
[174] (Ibid.)
[175] (Redhead, 58)

In this sort of theory a particle has to be treated either as a singularity in the field, or as a stable pulse of finite extent. The field from each centre decreases with the distance, but it never goes to zero. Therefore ultimately the fields of all the particles will merge to form a single structure that is an unbroken whole. (Bohm & Hiley 1993, *The Undivided Universe,* p. 352)"[176]

He continues: "It is not true, I have argued, that the material world is the totality of single things, substances, with a definite size each of which is somewhere in space and between which there is nothing. I have suggested that we better describe the world ontologically not as the totality of things or particles but rather as consisting of fields which are extended over all of space. *A priori* we may be inclined to think that the world consists of permanently existent particles, but we should be sceptical about this."[177]

This has lead some argue directly for "a novel field trope-bundle (FTB) ontology on which fields are composed of bundles of particularized property instances, called tropes"[178] whereby (i) particles are rejected or abandoned along with (ii) a substance ontology. We see how this might work given that "Field ontology does justice to the fact that there is the same structure everywhere by claiming that the world consists of fields which are extended over all of space, and it does justice to the fact that the world is different in different regions by claiming that fields have different intensities in different regions"[179] – that a field is in fact a sort of *sui generis* entity; and if each particle is a unique value in an 'unbroken whole' of values; and if multiple fields are overlaid over each other (and perhaps interact with each other) we can see how fields might play the role of properties, field values might play the role of tropes and spacetime events might play the role of bundles. If this is the case, and there is still much dissent and controversy over this, then we are perhaps lead down the path to reject a substance, and hence a particle, approach as an interpretation of QFT, and perhaps in metaphysics as well.

I will note here that this project is not as clearly evident as either side would like to think. Schneider has suggested: "A field approach to ontology has a sort of 'holistic character' and, thus, is alien to the usual ontological/conceptual parsings of the universe of discourse by means of (philosophical) categories and notions such as 'universal' and 'particular', 'substance' and 'accident', 'exemplification', 'instantiation', and the like. In light of the success of mathematical field theory within our best scientific theories, and because discontinuities between those theories and ontology should be minimized, it appears worthwhile to tackle the formidable ontological and meta-ontological task of rethinking the paradigms of and intuitions about ontological categories and kindred concepts that have, traditionally, guided ontological inquiry."[180] She continues: "Field spaces are neither universals nor particulars, *in the senses put forward here*. Field spaces are more than a collection of particulars. They are entities *sui generis,* internally complex, that capture general as well as particular aspects. The particular aspects are captured by a

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[176] (Wachter, 26)
[177] (Ibid.)
[178] (Wayne, 2)
[179] (Wachter, 26)
[180] (Schneider, 26)

field space's concretizations, and the general ones by the coordinating and holistic role the field space plays."[181]

What seems clear to me however, is that the ways by which philosophers have traditionally distinguished between entities are fast becoming obsolete. For example, the traditional substance ontology which forms the underlying basis for standard metaphysical or ontological physicalism is, in my opinion, archaic and in need of substantial revision regardless of whether or not FTB succeeds. The success of FTB just lends additional credibility to our replacement or rejection of substance metaphysics and physicalism.

Having outlined one strand in contemporary metaphysics that opposes the traditional substance ontology (and presumably substance, or ontological, physicalism as well), I will now open a more sustained attack on physicalism with the MUH fresh in mind.

It is pertinent at this time to return to some of the issues surrounding OR. As we have seen, OR denies the traditional category of object-hood and accepts structures alone into its ontology (under its strongest variant). ARS also dispenses with individually subsisting mathematical objects in favor of structures (though it retains abstract objects). ARS advances the pertinent thesis that a mathematical structure is an abstract entity existing outside of space-time. For OR it is not clear whether or not a physical structure is itself abstract:

> "Let's grant that physical structure exists – what is it? Is it just a description of the properties of entities? This leads to epistemic SR again. What makes these properties physical and not mathematical? That they're the properties of an entity? Again, we come back to the same old question – what is that (individual or non-individual)? If the entity is dissolved/reconceived then all we have is the structure and at this level the distinction between the mathematical and the physical may become blurred…"[182]

Some opponents of OR have challenged that this ambiguity arises from the structuralist metaphysics itself - "we have to distinguish clearly between a mathematical structure and a physical structure"[183] - on the grounds that in failing to establish a distinction they argue, OR dissolves or collapses the distinction between the physical and the mathematical, committing us to Platonism.[184] This is exacerbated by the recent interest in mathematical structuralism and the idea of *ante rem* structures. French and Ladyman make clear their stance: "we are not mathematical Platonists with regards to structures"[185] – they seem inclined to the *in re* variety.

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[181] (Schneider, 25)
[182] (French and Ladyman [1], 45)
[183] (Cao, 58)
[184] (Cao, 57)
[185] (French and Ladyman [2], 75)

In a more speculative moment, Shapiro considers the extension of ARS abstract mathematical structures to the ordinary and physical world: he stipulates that ordinary objects and ordinary structures are not freestanding structures in the way that formal mathematical structures are.[186] He explains: "beyond mathematics, there is more to reference and satisfaction than is captured in model theory. For ordinary language, model theory must be supplemented with accounts of reference and satisfaction."[187] So, on his account it seems that there will remain a distinction between mathematical and perhaps physical or even ordinary structure. French and Ladyman offer a trivial distinction between the physical and the mathematical: that there are more structures of the latter than of the former.[188] And, as we have seen in section six it isn't clear that ARS and OR can be linked together in a coherent or acceptable way.

Are these distinctions 'strong' enough to deny the accusation of Platonism? I suggest that until things are spelled out a little more concretely (pun not intended) that the structuralist's justification for these assertions is not immediately evident – it may be possible to develop a more nuanced Platonist-leaning mathematical structuralism that is compatible with OR (say an IRS motivated Platonism). Thus, I will reconstruct what we understand to be one of the essential arguments asserted by Cao, who has been a vocal opponent of OR[189]:

[8.0] The scientific realist understands the scientific activity to be discovering, exploring and describing a *physical* world.[190]

[8.1] This physical world is something over and above the mathematical models, entities, or theories which describe it. This requires the scientific realist must commit to entities that are separate from, and which they take to interpret, the (mathematical) models which describe them.[191]

[8.2] Ontic structuralism suggests that the world is just (mathematico-) structural which requires that (i) structures to be real and subsisting (mathematical) entities and (ii) that our theories are literally isomorphic to the world.

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[186] (Shapiro, 260)
[187] (Ibid.)
[188] (French and Ladyman [2], 75)
[189] as we understand it, Cao "himself appears to incline towards the epistemic form. Now Cao tells us that he is neither, that he sees his position as a 'third way' of some sort." (French and Ladyman [2], 76) – what I will refer to as the traditional scientific realist view (the arguments Cao presents, I feel, are the arguments that a traditional scientific realist would argue as well). We should note that he mentions "Both the ontic and epistemic versions of SR are philosophically unsatisfactory also because both of them smack of phenomenalism and instrumentalism" (Cao, 69) and "We realists don't have to suffer from such a defeat" (Cao, 76).; French and Ladyman describe Cao's 'third way' as that "which attempts to unite a form of 'entity realism' with Worrall's (epistemic) structural realism." (French and Ladyman [2], 76)
[190] "What actually happens in the development of science is not, as French and Ladyman assert, the dissolution of physical entities into mathematical structures" (Cao, 66)
[191] from: "on their attempt to dissolve physical entities into mathematical structures… it carries them away from scientific realism and towards Platonic idealism" (Cao, 57)

[8.3] Scientific practice suggests that a (mathematical) theory or model is interpreted by the physical, which is seen as being something more fundamental, or prior, than the (mathematical) theory or model.[192]

[8.4] [8.2] contradicts [8.0], [8.1] and [8.3].

[*] Thus, ontic structural realism cannot be offered as a realist program in the philosophy of science. [193]

What is relevant to me here is that Cao seems to either (i) identify scientific realism with physicalism (that they would entail each other)[194] or (ii) suggests that if one commits to scientific realism one must necessarily commit to physicalism (that one entails the other). My justification for this, is that the kind of 'physical entities' that are required to justify this (reconstructed) argument seem at least to require a commitment to some sort of traditional physicliasm. Now despite one's various commitments, I believe that these underlying assumptions are subject to deep criticism. I will argue that scientific realism and physicalism, in its most popular incarnation, in fact do not support each other. Further, there is the outstanding problem of just how we are to conceive of structures *as entities*. Following the metaphysical revision proposed by the ontic structuralists, surely *structures* are to be understood differently than *objects*, and the impact of these considerations might be worth further investigation.

Let B and A be structures. In what sense are we to take these structures to be different in kind? to be distinct? to be identical?

[8.5] We can distinguish between kinds of structure by noting the set-theoretic predicates that hold for the various entities under inquiry.

[8.6] In the common parlance, we can identify distinct structures *up-to-isomorphism*. If two structures are not isomorphic, then they are not the same structure.

[8.7] If two structures are isomorphic, then for all intents and purposes, they are the same structure.

In this way we can distinguish between kinds of structure: monotonic, rings, groups, commutative algebras; and between two structures: A and B are distinct iff $\neg(A \cong B)$.

I will argue here, that unlike with objects there is no way to distinguish between two structures' substances because there are no substances which attach to a structure. It, therefore, makes no sense to argue for a distinction to be drawn between two structures that are said to be isomorphic, for there is no extra non-structural substance, as per [8.7]

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192 "The major trouble is that by removing underlying physical entities from consideration, they also take an important task away from physicists and philosophers of physics, the task of interpreting mathematical structures in terms of physical entities. This move has the effect of preventing us from penetrating into the deeper layers of the physical world, and thus is counter-productive and even detrimental to the development of physics." (Cao, 77)

193 We derive this from the following passages, and the above: "In blending physical structures with mathematical ones, which paves the way for dissolving the former into the latter and for finally eliminating the former from the discourse…" (Cao, 58) and "[unlike] the ontic version: entities are not assumed to be dissolved into mathematical structures" my brackets; (Cao, 68)

194 he certainly identifies with some variant of scientific realism: "We realists…" ; (Cao, 76)

above. If one suggests that there is some non-structural component to the entities in the world, then their view collapses into ESR. So the proponent of OR appears obligated, in consideration of a variant of Platonism, to defend a non-isomorphic account of representation in order to save a distinction between the mathematical and the physical. However, I raise the observation that any weaker morphism is still a relationship between mathematical structures. If the proponent of OR were to take it that ‘physical structure’ is something over and beyond mathematical structure, in what sense do we say that the physical is structural then? For then the weight of the no-miracles argument, which under OR takes effect at the level of the mathematical edifice itself, would lead OR to the same criticisms raised against ESR.[195]

These, I feel, are important considerations when we look to the secondary aim of this paper. While, of course, the questions surrounding OR and ARS remain unsettled – I feel that it is pertinent to investigate this question given the presence of the Pythagorean heuristic identified in section one. So I ask this question: assuming OR and some Platonist inspired mathematical structuralism, in what way can we meaningfully, and non-trivially, draw a distinction between mathematical and physical structures?

To answer this question, and in order to more thoroughly lay out my criticism of physicalism, I will explore what I feel are Cao’s implicit assumptions regarding Scientific Realism, Naturalism and Physicalism.

9. Scientific Realism, Naturalism and Physicalism

I take realism, in general, to be (i) the position that those objects which are in the ‘domain of discourse of x’ are in fact ontologically significant and that these objects exist independently of the human mind and (ii) that statements made about those objects which are in the ‘domain of discourse of x’ either hold true or false of those objects thereby establishing a truth value account for x.[196] A second way to consider this is that a realist holds that the subject matter in question has a real ontological status and/or that ontological statements about the subject matter in question are not vacuous or fictitious. This is usually taken to mean that this subject matter is somehow “independent of anyone's beliefs, linguistic practices, conceptual schemes, and so on.”[197]

Thus, I take scientific realism to be that position which claims, hinging on the apparent power of the no-miracles argument, that (i) science is really discovering subsisting entities that exist in the objective world and (ii) that the predictive power and success of scientific theories (their approximate truth) are due to their latching onto these objective entities.

The compelling suggestion has been made by a number of commentators that many philosophers conflate scientific realism with physicalism, and also naturalism with

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[195] recall the criticism’s levied against Russell’s distinction between first and second order structure

[196] I argue that ‘objects’ track, or pick-out, real patterns and hence, can meet the ontological conditions for realism outlined here; see (Ladyman and Ross, 220-34)

[197] (Miller)

scientific realism (and thus the three are taken to be somewhat interchangeable or equivalent). I argue that, following the pessimestic meta-induction it cannot be the case that scientific realism just equates to physicalism (consider the demise of physicalism's predecessor: materialism), nor that scientific realism just means naturalism. I offer a treatment of this notion:

[9.0] I define a metaphysical position 'm' to be compatible with naturalism if and only if 'm' is motivated by (or derived from/compatible with) a contemporary (best) scientific theory.

[9.1] A contemporary scientific theory is a best scientific theory at some time t.

[9.2] A best scientific theory is a scientific theory enshrined into scientific practice and institutions as the premier way to explain, predict or describe the behavior of observed phenomena in a particular area of scientific investigation (e.g. psychology, physics, microeconomics).

My justification for this rests on the grounds that I take naturalism to mean that we should be motivated to constrain our philosophical activities, at least those that wish to make the claim toward objective truth, by science; and that science through a process of continual revision and theory change continues to reject certain theories or to produce better ones. Thus, any metaphysical position that fails to derive from a current best scientific theory likewise fails to be a metaphysical position motivated, or compatible with, naturalism.

Now, I will characterize physicalism as being any particular metaphysical position which satisfies (holds) these assumptions:

[9.3] The world is comprised of levels.

[9.4] There is a fundamental level.

[9.5] This fundamental level is a collection of irreducible atoms.

[9.6] These atoms interact locally (micro-banging).

[9.7] These atoms are objects (i.e. individuals) which are understood to be something to which properties attach to (bare particulars) and/or which display a primitive essence, *haecceity*, *quiddity* or substance.

[9.8] These objects are physical.

I now argue that physicalism, so conceived, fails to be motivated by a best scientific theory and is thus not compatible with naturalism. By the definition outlined for naturalism above, I set the necessary and sufficient conditions for a metaphysical position to be compatible with naturalism. I argue that this requires that the metaphysical position in question be in correspondence with a current best theory of some scientific domain. Note that physicalism is motivated to provide a realist metaphysics or ontology for natural metaphysics, presumably derived from physics itself.

I need only turn to Quantum Mechanics to explicitly falsify condition [9.6] through quantum entanglement. Furthermore, [9.3] is reliant on [9.4] and [9.5] neither of which are necessarily true or given by empirical observation.[198] From our considerations in section five I am inclined to doubt [9.8], if not to reject it all-together. Thus,

[9.9] Physicalism is motivated from Classical Mechanics and Physics.
[9.10] Classical Mechanics and Physics are subsumed by Quantum Mechanics and Field Theory.[199]

Following these premises and conclusion it follows that physicalism stands in opposition with naturalism. Embarrassingly, many physicalist metaphysics do *not* make *any* reference to contemporary physics.[200]

10. Towards Pythagoreanism

Following my rejection of physicalism (and with the introduction of the metaphysical notion of 'physical structure'), I am then saddled with the burden of providing an explanation of just what 'physicality' amounts to. I will admit that I opt for something similar to the Physical Theory Account of Physical Objects following Markosian's classification:

> "One natural response to the question *What are physical objects?* is simply that physical objects are the objects studied by physics. I will call this the Physical Theory Account of Physical Objects. One main problem with the Physical Theory Account is that it seems likely to lead to circularity, since it's natural to want to define physics as the study of physical objects and the laws of nature governing them… In addition to this main problem facing the Physical Theory Account, there are several other objections that can be raised against that account. Here's one. Various abstract objects, like numbers, equations, and functions, not to mention other more obscure mathematical entities, are studied by physics. So the Physical Theory Account seems to entail that these objects should count as physical objects. But they shouldn't."[201]

I do agree that to say 'physics is the study of physical objects and laws' and then to hold the position that 'physical objects are the objects studied by physics' is indeed circular. Replacing 'objects' with 'entities' gains no traction either.

Before attempting to provide an answer for this I will note that there have been some objections raised against giving priority to certain scientific activities[202] and thus against taking the heavy formalization endemic in certain scientific fields to be indicative of say the Pythagorean heuristic. To defend my view then, I must attempt at least to defend the primacy of physics. I am motivated to this position by the following:

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[198] (Ladyman and Ross, 20, 55-7)
[199] (Ladyman and Ross, 25)
[200] (Ladyman and Ross, 10, 18)
[201] (Markosian, 379)
[202] see again (Cartwright)

> "The first argument is inductive: in the history of science a succession of specific hypotheses to the effect that irreducibly non-physical entities and processes fix the chances of physical outcomes have failed."
>
> "The second argument is also inductive. Over the history of science a succession of processes in living systems, and in the parts of some living systems dedicated to cognition, have come to be largely or entirely understood in physical terms, by which we mean in terms of the same quantities and laws as are invoked in physical theorizing about non-living systems."[203]

Leading to the view that: "Special science hypotheses that conflict with fundamental physics, or such consensus as there is in fundamental physics, should be rejected for that reason alone. Fundamental physical hypotheses are not symmetrically hostage to the conclusions of the special sciences."[204]

The criticisms that have been levied against the primacy of physics often reference problems with either supervience or reductive physicalism. As I have argued in section ten, such notions ought to be dispensed with – that pseudo-scientific talk about levels[205] is merely an anachronistic artifact dating back to the 16th and 17th centuries. Hence, the formalization endemic in physics is a consideration that I feel cannot be ignored.

However, I want to criticize Markosian's implicit commitment to the Principle of Exclusion (he provides no other account for why "they shouldn't") which I take to be the traditional handle for talking about (and dividing reality between) the mathematical and the physical.

When one couples the idea that many working scientists are guided by the attempt to discover 'natural laws' (which rings of Platonism in that these laws are somehow taken to be prior to, or shaping of, the physical world) and recent theories of everything which are explicitly pythagorean in nature[206] we can find it appears that the traditional divisions between physical/abstract or physical/mathematical have already been narrowed.[207]

I remind the scientific realist of the Quine-Putnam Indispensability Argument:

> "(P1) We ought to have ontological commitment to all and only the entities that are indispensable to our best scientific theories.
>
> (P2) Mathematical entities are indispensable to our best scientific theories.

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[203] (Ladyman and Ross, 43)
[204] (Ladyman and Ross, 44)
[205] see (Ladyman and Ross, 20-1, 53-7)
[206] consider Stoica "The world is defined as a mathematical structure containing the spacetime, which in general is a topological space, and the physical laws, expressed as a sheaf over the spacetime." (Stoica, 2); see also (Lisi)
[207] see (Resnik)

> (C) We ought to have ontological commitment to mathematical entities."[208]

Taking contemporary scientific practice (particularly in the more formal sciences: physics, computational sciences, economics) into account, we argue that the Quine-Putnam Indispensability Argument gains greater weight. This is doubly improved by the ascendancy of the semantic view of scientific theories.[209]

Following the definition of realism offered above, I define mathematical realism to be the thesis that mathematical objects are real and exist independently of the human mind and that mathematical statements are about those objects and are therefore true or false in virtue of those objects.[210] Mathematical Platonism is presumably the current reigning mathematical realism. Mathematical Platonism usually makes three claims**:**[211]

[10.0] Mathematical objects (mathematicalia) exist.
[10.0] $\exists x(Mx)$[212]
[10.1] All mathematicalia are abstracta (all mathematical objects are abstract).
[10.1] $\forall x(Mx \rightarrow Ax)$
[10.2] Mathematicalia are independent of the physical world and its intelligent agents.
[10.2] $\forall x(Mx \leftrightarrow \neg Px)$

I take [10.1] to be an uncontroversial thesis.[213] [10.0] is usually defended by appealing to semantic accounts of truth and reference – i.e. the traditional arguments levied against psychologism (raised perhaps most famously by Frege). As noted in section seven, mathematical practice also suggests, at least superficially, existence claims which are then used in foundational axiomatic systems which produce statements that are understood to be true.

Before turning to [10.2], a more thorough defense of [10.0] ought to be offered. A common strategy employed by mathematical realists in order to defend the respectability of [10.0] under a naturalist metaphysics was to appeal to the Quine-Putnam Indispensability Argument. Recently, Harty Field, who has nominalist leanings, has attempted to outflank the Quine-Putnam Indispensability Argument by demonstrating that numbers are in fact dispensable to scientific activity. [214] For Field, numbers are primarily fictitious entities which are, in theory, replaceable by concrete physical or ordinary objects whose status we have fewer dubious reasons to take as ontologically significant. These efforts have been, of course, met with great criticism. I do not have time to go into the specifics of this proposal here but I will note that while Field's attempt

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[208] (Colyvan)
[209] (Brading and Landry [1], 5)
[210] more technically this might be considered realism in ontology and truth value, respectively
[211] working from the definitions set forth in (Linnebo [3])
[212] I use these first-order sentences just for convenience, presumably we are ranging over all entities/objects with our quantifiers.
[213] the early Maddy is a noted exception, see (Maddy[1])
[214] see (Field)

has been much celebrated by antirealists, most observers do not seem to think that his project has succeeded in its goal of dispensing with numbers in science.

A recent defense of [10.0] has been offered by Tennant. I will sketch out a small portion of his methodical defense here. First, Tennant introduces Hume's Principle which has been defended by Hale and Wright,[215] and which was "regarded most famously by Frege,"[216] as being analytic. Tennant denotes this as *Schema (N)*:

> "The number of Fs is identical to the number of Gs if and only if there are exactly as many Fs as Gs."[217]

For Tennant *Schema (N)* ought then to be replaced by *Schema (C)*: "the number of *F*s is <u>*n*</u> if and only if there are exactly *n F*s."[218] *(C)* avoids many of the difficulties facing Hume's Principle: "Arguably, *(C)* is a philosophically deeper condition than the Humean identity *(N)*, at least for finite numbers; since for these *(C)* implies *(N)*. Significant also is the fact that *(C)* is completely expressed (albeit schematically) at first order:

> #*xFx* = <u>*n*</u> if and only if there are exactly *n F*s

once granted that the variable-binding term-forming operator #xΦx is a first order expression. By contrast, (*N*) involves second order quantification on at least one side, insofar as it deals with the equinumerosity of concepts *F* and *G*."[219]

Following Tennant we are then lead, with additional insights, to the conclusions that:

> "In any world in which one uses a rich enough first-order language with the identity predicate, the existential quantifier, negation and the numerical term-forming operator #—one has (on reflection) to acknowledge the existence of zero. For in any such world there are no things that are not self-identical; whence 0 is the number of such things; whence 0 exists."[220]

> "The question being considered is whether 0, *qua number*, exists. Our answer is that it does. 0 is a very special number; it is the number of any empty concept-in particular, the number of things that are not self-identical. Expressed more formally, 0 is #x(¬x = x)."[221]

> "The conception of analyticity at work here is one that is shorn of the usual dogma that an analytic statement can involve no ontological

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[215] (Tennant, 311)
[216] (ibid.)
[217] (ibid.)
[218] (Tennant, 316)
[219] (Tennant, 318)
[220] (Tennant, 321)
[221] (Tennant, 322)

> commitments. It is analytically true that $0 = \#x(\neg x = x)$, and this in turn implies that 0 exists."[222]

Leading us to the *necessary* existence of the natural numbers:

> "To grasp the meaning of "0" is to know what number it denotes, not merely to know that it would denote such-and-such a number provided only that that number existed. This line of argument, followed through, leads one also to the necessary existence of the natural numbers, at least if necessity is construed as truth in every possible world about which predicative and quantificational thought is possible, or within which such thought is possible about it. For, once such thought is possible, so is its extension (if it is not yet extensive enough) to thought about numbers…The necessary possibility of doing so guarantees the necessary existence of the natural numbers."[223]

I shall now return to address [10.2] above. Clearly [10.2] divides the physicalist and Platonism camps – not only in the metaphysical sense of forming the basis for the metaphor found in 'a mathematical reality separate or prior to the physical world' but also in the sense that nominalists tend to champion the physicalist side opposed to the Platonists. I add further that the epistemological arguments against the existence of mathematicalia, and indeed the motivation for most antirealist (in mathematics) claims, at least on my view, seem to stem from [10.2].

It is my belief that this division can be dissolved into an even more fundamental set of distinctions. Let these more fundamental distinctions be called the 'Abstract/Concrete Distinction' which I will identify by the moniker 'the Principle of Exclusion':

[10.3] No abstract thing is concrete. No concrete thing is abstract.
[10.3] $\forall x(Ax \leftrightarrow \neg Cx)$
[10.4] There is nothing that is both concrete and abstract.
[10.4] $\neg\exists x(Ax \wedge Cx)$
[10.5] All physical things (physicalia) are concrete.
[10.5] $\forall x(Px \rightarrow Cx)$

Adding to these the uncontroversial thesis that:

[10.6] Every entity is either abstract, concrete or both (when coupled with the Principle of Exclusion we derive the standard dichotomy).
[10.6] $\forall x(Ax \vee Cx)$

So upon investigating [10.2] it would seem that [10.2] is partly the consequence of those premises which comprise the Principle of Exclusion [10.3], [10.4] and [10.5] above]: For instance we see immediately how [10.2] follows from:

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[222] (Tennant, 325)
[223] (Tennant, 325-6)

[10.0] $\forall x(Ax \leftrightarrow \neg Cx)$
[10.1] $\forall x(Mx \rightarrow Ax)$
[10.5] $\forall x(Px \rightarrow Cx)$

I take it that to justify [10.2], at least axioms [10.0], [10.1] and [10.5] would have to be taken – i.e. that [10.2] is a maintainable position given the Principle of Exclusion. However, without these axioms I see no reason to adopt [10.2] particularly in the light of the considerations raised in sections three and one.

We also see that without the axioms which constitute the Principle of Exclusion, [10.2] loses its analyticity. So to knock out [10.2], it is necessary only to knock out [10.2] and not the other axioms [10.0] and [10.1] of mathematical Platonism. I think that it is sufficient to note that the Principle of Exclusion is not something known *a priori* (just note the disagreement between Plato and Pythagoras), nor does it seem to be a necessary truth. So, like Pythagoras, I would like to see a metaphysics that enables a continuity between the abstract and the concrete.[224] This has been suggested by numerous others, as well: “The change Heisenberg proposes suggests that there is more continuity between the ontology of mathematics, the ontology of physics, and the ontology of the objects of our everyday experience than many philosophers have believed there to be…”[225]

I argue that, following these considerations, we should reject the Principle of Exclusion. Thus, I propose that characterizing the physical as being opposed to the abstract is unmotivated, as is the third premise of (mathematical) Platonism. In order to provide a better account for just what physicality amounts to, I feel it prudent to take into consideration at least two interesting notions: on the one hand scientists routinely employ, or develop new mathematical systems for employment from pre-existing, mathematical systems (and, perhaps, abstract entities) in order to describe the physical; on the other hand mathematicians may often work from observed phenomena in the world to develop a new mathematical system or to extend previously existing ones.[226]

And, tentatively, I feel that the notion of modeling measurements and/or data plays a critical role in linking mathematical models to the world. To answer Markosian, I will suggest here that what qualifies a particular structure as ‘being physical’ is that it is (i) a mathematical structure that (ii) accurately models a set of measurement data, and (iii) is our current best and simplest way to do so.

Once we have dispensed with the ‘exclusive’ set of binary distinctions between the abstract/concrete and between the mathematical/physical it follows that a number of other

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[224] Consider also the problems that surround the traditional characterization of the concrete: the traditional way to separate entities into either the ‘piles’ concrete or abstract is by appealing to spatio-temporal locateability (i.e. that entity’s location in spacetime). Couple this with the traditional assumption [10.5] and we run into the absurd problem of not being able to see spacetime itself as physical!
[225] (Hale, 383)
[226] Consider the introduction of matrices by Heisenberg, Born and Jordan which played a vital role in the establishment of quantum mechanics.

metaphysical assumptions ought to be revised. For brevity's sake I will merely provide a graphical representation of the new metaphysical view I am inclined to accept.

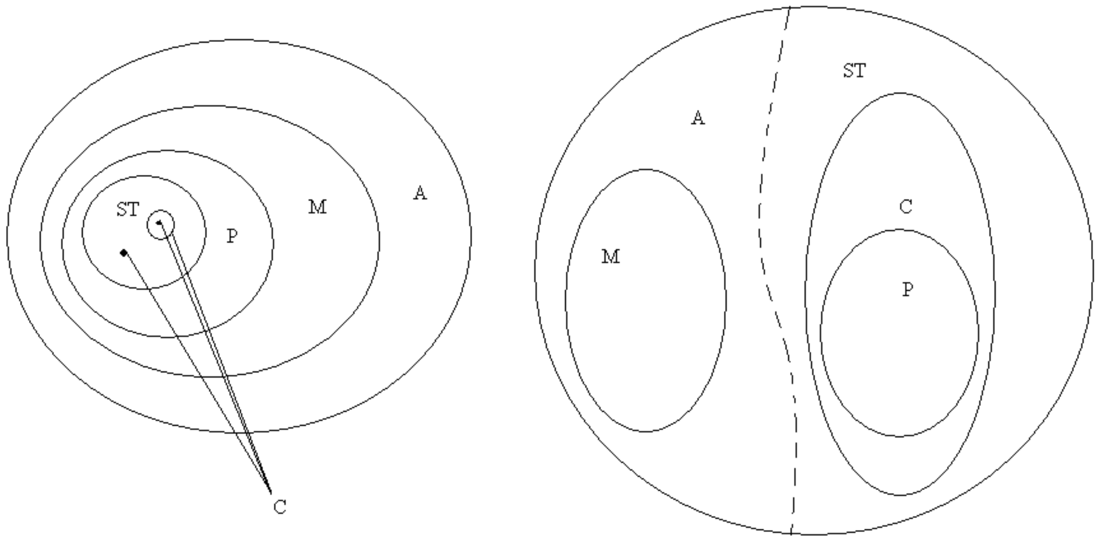

Figure 3. A graphical representation of something like the view proposed in this paper and a representation along the lines of the more traditional set of distinctions.

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