V. Formal Mereology

[Characteristics]:

[1] First-order.
[2] Classical.
[3] Complete.
[4] Consistent.

PP
O
U
P

[$*_1$]	Parthood	$Pxy \leftrightarrow \forall z[Ozx \rightarrow Ozy]$
[$*_2$]	Proper Part	$PPxy \leftrightarrow (Pxy \wedge \neg Pyx)$
[$*_3$]	Overlap	$Oxy \leftrightarrow \exists z[Pzx \wedge Pzy]$
[$*_4$]	Underlap	$Uxy \leftrightarrow \exists z[Pxz \wedge Pyz]$

[A_1]	Reflexive	Pxx
[A_2]	Antisymmetric	$(Pxy \wedge Pyx) \rightarrow x = y$
[A_3]	Transitive	$(Pxy \wedge Pyz) \rightarrow Pxz$
[A_4]	Weak Supplementation	$PPxy \rightarrow \exists z[Pzy \wedge \neg Ozx]$
[A_5]	Strong Supplementation	$\neg Pyx \rightarrow \exists z[Pzy \wedge \neg Ozx]$
[A_6]	Atomistic Supplementation	$\neg Pxy \rightarrow \exists z[Pzy \wedge \neg Ozy \wedge \neg \exists v[PPvz]]$
[A_7]	Top	$\exists x \forall y[Pyx]$
[A_8]	Bottom	$\exists x \forall y[Pxy]$
[A_9]	Sum	$Uxy \rightarrow \exists z \forall v[Ovz \leftrightarrow (Ovx \wedge Ovy)]$
[A_{10}]	Product	$Oxy \rightarrow \exists z \forall v[Pvz \leftrightarrow (Pvx \wedge Pvy)]$
[A_6]	Unrestricted Fusion	Where [$\phi(x)$] is a first-order wff and x a free variable: $\exists x[\phi(x)] \rightarrow \exists z \forall y[Oyz \leftrightarrow \exists x[\phi(x) \wedge Oyx]]$
[A_{12}]	Atomicity	$\exists y[Pyx \wedge \neg \forall z[PPzy]]$

Notes

[A_{11}] entails [A_7]
[A_8] is controversial

Systems

*****	$\{*_1, *_2, *_3, *_4\}$
M	$* \cup \{A_1, A_2, A_3\}$
MM	$\mathbf{M} \cup \{A_4\}$
EM	$\mathbf{M} \cup \{A_5\}$

CEM	**EM** $\cup$ $\{A_9, A_{10}\}$
GM	**M** $\cup$ $\{A_{11}\}$
AGEM	**M** $\cup$ $\{A_6, A_{11}\}$